Advanced Quantum Mechanics and Path Integrals

Lecture Course by
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34 hours, 17 Lectures.

1. **General concepts of quantum mechanics**
   Introduction: Stern and Gerlach (SG) experiment. Magnetic moment of electron. Successive experiment of SG. Analogy with light polarization. ***Problems.***

2. **Mathematical apparatus of quantum mechanics**
   Ket- and Bra- spaces and vectors (state vectors). Physical operators and observables. Eigenvectors and eigenvalues. Basic vectors and matrix representation. Hermitian conjugate and Hermitian operators. Self-adjoint operators. Distinction from Hermitian ones. Superposition principle. Completeness relation. ***Problems.***

3. **Principle postulates of quantum mechanics**
   Measurements, observables. Notion of state probability. Mean (average) values of operators. Compatible and incompatible observables. The uncertainty relation. Unitary operators. Unitary transformations, change of basis, unitary equivalent observables and their properties. Transformation matrices, their diagonalization. ***Problems.***

4. **Fundamental operators of quantum mechanics**
   Space- translation. Generator of translation. Momentum, as translation generator. Properties of momentum operator, its hermitivity. Commutator of coordinate and momentum operators. Heisenberg uncertainty relation between them. The eigenvalue and eigenfunction of momentum operator (in coordinate and momentum representations). The question of self-adjointness of momentum operator. ***Problems.***

5. **Theory of angular momentum**
   Angular momentum operator as a generator of 3-dimensional rotations. Finite and infinitesimal rotations. *Exercise*: angular velocity as a vector. Explicit form of angular momentum in coordinate representation. Commutation relations for

angular momentum. Momentum operator in spherical coordinates. Square of angular momentum and its Z-component. Their eigenfunctions and eigenvalues. Angular momentum as a quantum operator. ***Problems.***

6. **Time evolution and quantum dynamics**

The evolution operator. Hamiltonian. Operator of Unitary transformations and the Schrodinger equation for it. Time-dependent and time-independent Schrodinger equations. Energy-time uncertainty relation, its interpretation. Various representations of the Schrodinger equation – Schrodinger versus Heisenberg picture, Dirac picture. Formal Dyson series. ***Problems.***

7. **Short survey of quantum mechanical equations**

Matrix elements of the unitary operators and transformations. Matrix element of the Unitarity transformations, as a Green's function of the Schrodinger equation. The Huigence principle. The Green function in various representations. ***Problems.***

8. **Phases, potentials and gauge transformations**

Constant potentials. Gravity in quantum mechanics. Gravity induced quantum interference. Monoenergetic beam of particles (thermal neutrons) and gravity induced phase (observation). Gauge transformation in electromagnetism. Aharonov-Bohm effect.

9. **Path integral and quantum mechanics**

Coordinate and momentum operators in a self-representation, transition between them, as a Fourier transform. Transition to quantum mechanics and the problem of operator's ordering. Normal and Weil orderings. Baker-Hausdorf- Kembpell relation. Calculation of transition probability. Preliminary form of Feynman's path integral. Integration by momentum when Hamiltonian is a sum of terms depending on coordinates and momentum separately. The final form of Feynman path integral. ***Problems.***

10. **Geometrical and Physical interpretation of the path integral**

Classical limit of the path integral. Equivalence to Schrodinger description. Path integral in case, when coordinate and momentum are not separable into the Hamiltonian. Effective action (Lie and Yang method). ***Problems.***

11. **The simplest cases of path integral calculation**

The free particle, general square form. Harmonic oscillator by Furrier transform. Calculation of classical action be the Green function method. Green function's poles and methods of their remove. Feynman prescription. Spectrum of the harmonic oscillator. Three dimensional case – path integral in the radial variables. ***Problems.***

13. **Change of variables in path integral**

    The point canonical transformations in operatorial formalism. Weil ordering in the path integral. Canonical transformations. Space-time transformation in path integral. Transformation's explicit form. Derivation of transformations by stochastic approach. Relation between Feynman kernels after transformation. Some solvable problems – Coulomb problem, valence electron potential, other potentials. ***Problems.***

14. **Second quantization**

    Schrodinger quantum mechanics as a first quantization. Quantized of wave function by first-quantization method. Solution of principal equation by operators. Two form of solutions – quantization of wave function by commutators and anti-commutators. Two possible statistics. Harmonic oscillator in the second quantized version: creation and annihilation operators. Number operator. Positivity property, bounding of number operator from below. Definition of vacuum state. Construction of spectra by acting of creation operator on vacuum ket. Normalized kets. Derivation of wave functions by operator method.

15. **Supersymmetric (SS) quantum mechanics (QM)**

    Motivation – boson-fermion symmetry. Generators and SS Hamiltonian. Witten's algebra. Structure of superalgebra. Non-negativity of Hamiltonian spectrum. Two-fold degeneracy for non-zero energy. Charges and supercharges. Non-usual properties of transformation parameters. Grassman's algebra, Grassman's numbers. Zero energy of vacuum. First example of SS in real world. SSQM. Matrix realization of fermionic (odd) operators. Hamiltonian and superpotential. ***Problems.***

16. **Theory of Grassmanian variables**

    Global properties of SS Hamiltonian. Berezin formalism – differentiation and integration by Grassman variables. Holomorphic representation for bosons. Holomorphic representation for fermions. Integrals in holomorphic picture. Berezinian. Gauss integrals. ***Problems.***

17. **Elements of Grassmanian analysis**

    Basis of Grassman numbers. Nilpotent operators. Taylor expansion of Grassmanian functions. Linear space $G_n$. Examples of application of grassmanian integrals. Fermion classical functions expressed by Grassman numbers. Feynman integral for fermions. ***Problems.***

***In progress:*** *Quantization of Gauge theories.*