UNIVERSITY OF L'AQUILA

DEPARTMENT OF PHYSICAL AND CHEMICAL SCIENCES

Master's Thesis in Physics

# Grand Unification of Fermion Masses

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# 1 Introduction

Nowadays, Standard Model is the most successful model that is tested by many precise experimental measurements. **SM** coalesces in itself many fundamental theories, like: **Quantum Field Theory** that provided ultimate view of describing particles and their interactions; **Dirac equation** that describes dynamics of spin $\frac{1}{2}$ particles-fermions and of course **gauge principle** that gives us clear description of interaction between gauge and fermion-boson fields. Ultimately, SM is nothing without the **Higgs Mechanism**, that with its brilliant method for breaking the electroweak symmetry generates masses. However, there are some uncertainties and problems. Standard Model of particle physics has 26 free parameters, we insert it on our own. There are twelve masses of fermions, three coupling constants describing gauge interaction, two parameters for Higgs potential eight mixing angles and one from CP violation.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $m_e$ | $m_\mu$ | $m_\tau$ | $m_{\nu_1}$ | $m_{\nu_2}$ | $m_{\nu_3}$ | $m_u$ | $m_c$ | $m_t$ | $m_d$ | $m_s$ | $m_b$ | $\theta_{cp}$ |
| $g'$ | $g_W$ | $g_s$ | $v$ | $m_H$ | $\theta_{12}$ | $\theta_{13}$ | $\theta_{23}$ | $\delta$ | $\lambda$ | $A$ | $\rho$ | $\eta$ |

These 26 parameters are chosen in a way to match the observations. It does not come from a higher theoretical phenomena. In other way, we can say that - theory cannot fix them by itself. Moreover, it should be said that SM describes only 3 fundamental forces Strong, Electromagnetic and Weak. Therefore, Gravity is somewhat "oppressed".

Despite this huge problem SM gives us some hints. If we look on the pic.1 we can see that masses of fermions are divided by family. It is clear that in each family fermions have masses of the same order, this is unlikely to happen by chance. Consequently, we might think that our Standard Model might be some low-energy manifestation of some, more general theory.

Such hierarchy between the fermion masses is highly intriguing for us. We do not know why the spectrum is spread from MeVs to GeVs (nearly $10^5$ order). In SM Lagrangian Yukawa terms are vital to give particles masses after SSB and Highs mechanism. These Yukawa matrices are proportional to the mass matrices and they are arbitrary. After some procedures, diagonalizing and getting eigenvalues we get small mixing angles between up and down quarks. So, our problem can be stated in a different way: why up and down quark matrices are aligned in a way to give us small angles?

To explain this phenomena, it is appealing to consider some scale between electroweak and Plank. Then we assume that there exists more fundamental theory that can make some constraint on Yukawa terms.

Nowadays the most promising ideas beyond SM are related to the concepts of GUT and SUSY. At some energy $SU(3) \times SU(2) \times U(1)$ is unified

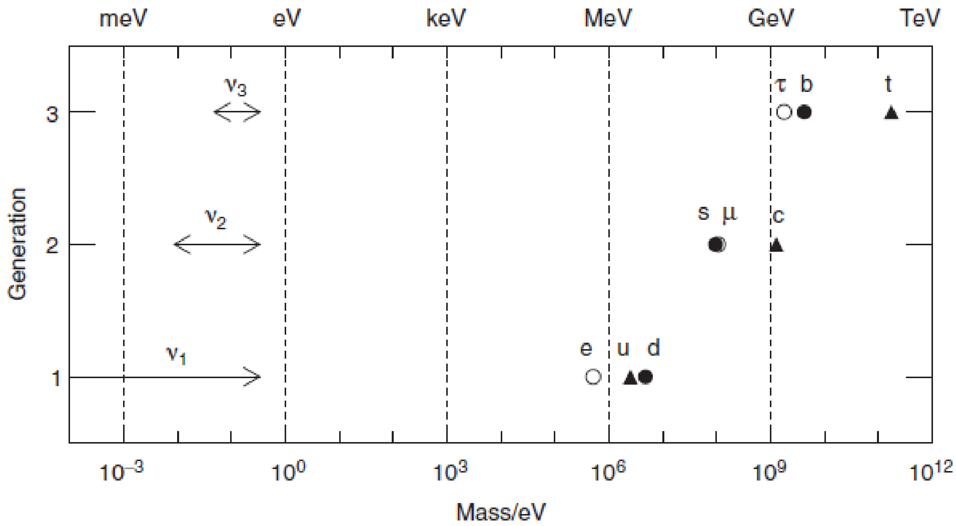



Figure 1: The fermion masses shown by generation. Here, the neutrino masses are displaye as approximate ranges of values assuming that we have normal hierarchy. The upper limits on the sum of neutrino masses from cosmological constraints are being used [7].

into the $SU(5)$. Is $SU(5)$ unification the answer? unfortunately, no. It can provide vital improvements, however Yukawa terms $Y^u$ and $Y^d$ still remain arbitrary. So, we are forced to go beyond SU(5) and implement new ideas that can shed more light on our problem.

Creating a model is like playing a lottery, because you do not know at the end if you win or not(if you get correct results or not) therefore we tried different models and schemes and at the end we got the results with high precession. Firstly, we are working of a GUT scale considering SU(5) symmetry, using its' perks and algebra. However, to explain hierarchy between masses we say that there exist new symmetry between families that is called a horizontal symmetry $SU(3)_H$ that is chiral and compatible with grand unification. It is impressive that by writing operators of processes we can make realization of Fritzsch-like matrices where we have three free parameters initially, A,B and $\alpha$. Then comes the fourth one from Clebsch coefficients. Our task was to fix these parameters from knowing lepton masses and their divisions however, how unfortunate it sounds $SU(5) \times SU(3)_H$ did not gave us physically correct model. Consequently, we went a little bit "up" and introduced $SO(10)$ that made problem more symmetric. It was quite impressive how this change made task more united and clear. We made realization of Fritzsch-like mass matrices and finally got brilliant results for quarks. We used constantly iteration method during calculation to make results more and more accurate. On a table below you can see what was our input and what we have got:

| **Free Parameters** | | | |
|---|---|---|---|
| $A$ | $B$ | $\alpha$ | $x$ |
| Input in the Model: **Whole Lepton Part+(23) mixing angle** | | | |

| **Output** | | |
|---|---|---|
| $\frac{Y_u}{Y_c} \sim \frac{1}{500}$ | $\therefore$ | $\frac{Y_d}{Y_s} \sim \frac{1}{20.3}$ |
| $m_u(2GeV) \simeq 2.64\ MeV$ | $m_c(M_c) \simeq 1.32\ GeV$ | $m_t(M_t) \simeq 163.42 GeV$ |
| $m_d(2GeV) \simeq 4.768\ MeV$ | $m_s(2GeV) \simeq 96.8\ MeV$ | $m_b(M_b) \simeq 4.117\ GeV$ |
| $s_{12} \simeq 0.22$ | $\therefore$ | $s_{13} \simeq 4 \cdot 10^{-3}$ |

As you can see all values fall into the error range of pdg values [9].

# 2 Standard Model

## 2.1 Overview

For the Standard Model we have local symmetry group:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \tag{1}$$

Here C stands for the color charge. L and Y, left-handed chirality and weak hypercharge respectively. We have to have Lorentz invariance, gauge invariance, therefore mediator of the interactions are bosons of their group, that have their own coupling constants. About $SU(3)_C$ we can say that it is the non-abelian local symmetry group for QCD. It has $N^2 - 1$ generators, that describe eight massless gluons. Electromagnetic and weak interaction is unified into electroweak (EW) $SU(2)_L \times U(1)_Y$ . Mediators are 3 massive particles, massive bosons ($W^{\pm}, Z^0$) and the massless photon $\gamma$.

Fermions are two types, quarks and leptons. Both are divided into three families, generations. Up, Charm and top quarks have charge $+\frac{2}{3}$ and Down, Strange, Bottom quarks have $-\frac{1}{3}$. In leptons we have Electron, Muon and Tauon that have charge $-1$ and three chargeless neutrinos.

| **Standard Model** | | | | |
|---|---|---|---|---|
| Three Generations of Matter | | | Interactions/force carriers | |
| $1st$ | $2nd$ | $3rd$ | Bosons | |
| $u^{+\frac{2}{3}}$ | $c^{+\frac{2}{3}}$ | $t^{+\frac{2}{3}}$ | g | H |
| $d^{-\frac{1}{3}}$ | $s^{-\frac{1}{3}}$ | $b^{-\frac{1}{3}}$ | $\gamma$ | |
| $e^{-1}$ | $\mu^{-1}$ | $\tau^{-1}$ | $Z^0$ | |
| $\nu_e^0$ | $\nu_\mu^0$ | $\nu_\tau^0$ | $W^{\pm}$ | |

We construct theory in a way that left-handed and right-handed chiral components of the fermions "construct" different representations. So for quarks we can say that their left-handed components are grouped into weak isospin doublet and color tiplet. Leptons are singlets of $SU(3)_C$ but doublets of $SU(2)_L$. RH components are just weak isospin singlets. This can be summarized on the diagram below [7]

| | | $I$ | $I_3$ | $Y$ | $Q$ |
|---|---|---|---|---|---|
| Lepton Doublet | $l_L \equiv \begin{pmatrix} \nu_L \\ l_L \end{pmatrix}_i$ | $\frac{1}{2}$ | $\frac{1}{2}$ <br> $-\frac{1}{2}$ | $-\frac{1}{2}$ | $0$ <br> $-1$ |
| Lepton Singlet | $l_{R_i}$ | $0$ | $0$ | $-1$ | $-1$ |
| Quark Doublet | $q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}_i$ | $\frac{1}{2}$ | $\frac{1}{2}$ <br> $-\frac{1}{2}$ | $\frac{1}{6}$ | $\frac{2}{3}$ <br> $-\frac{1}{3}$ |
| Quark Singlet | $u_{R_i}$ <br> $d_{R_i}$ | $0$ | $0$ | $\frac{2}{3}$ <br> $-\frac{1}{3}$ | $\frac{2}{3}$ <br> $\frac{-1}{3}$ |
| Higgs Doublet | $\Phi \equiv \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$ | $\frac{1}{2}$ | $\frac{1}{2}$ <br> $-\frac{1}{2}$ | $\frac{1}{2}$ | $1$ <br> $0$ |

## 2.2 SM Problems

In SM there are plenty of incompleteness, which are known as SM problems. For example, the hierarchy problem: The **Higgs** mass is $m_H \approx 125GeV$ whereas the gravitational scale is $M_{Plank} \sim \sqrt{G} \sim 10^{19}GeV$ So,firstly, the "Hierarchy Problem" stands as: why do we have $m_H/M_{plank} \sim 10^{-17}$ so small? This question arises because in fundamental theory we expect them to be the same. The hierarchy is stable with respect to the quantum corrections so this problem is called **technical hierarchy problem** and it is the motivation for weak-scale super-symmetry. Secondly, the cosmological constant problem that with respect to the plank mass is really small. we have $(\lambda/M_{plank})^4 \sim 10^{-120} \ll 1$ Moreover, there exists some open questions like which particles constitute into the CDM (cold dark matter). Large scale structure data favour a cosmological constant-cold dark matter model where nearly 22% of the universe's energy lies into the DM, only 4% in ordinary matter and 74% in the dark energy. Neutrinos are relativistic when they decouple from the thermal plasma therefore they constitute a hot component of our dark matter. It should be said that the SM and Gravity do not get together. Simply adding a graviton to the SM does not coincide with the experimental measurements. Also, neutrino mass problem is quite important. From neutrino oscillation model we know that they have mass however, SM cannot give it to them without "help". We put mass terms for neutrinos by hand that leads to a new theoretical problem. For example, the mass terms need to be small and it is not clear if the neutrino masses would arise in the same way that the masses of other particles. One of the problem which draws attention is a Strong CP problem. Theoretically SM should contain a term that breaks CP symmetry which is connected to the matter antimatter difference. However, it predicts that matter and antimatter should have been created equal amounts. Ultimately, we can say that SM is not perfect model and we should look for something more complete and precise. [7]

## 2.3 Strong Interaction

Group that describes strong interaction is called $SU(3)$. It has eight generators that correspond to 8 massless gluons. Six quarks and six antiquarks are represented in three colors Red Blue and Green. Gluons carry color charges and are mediators of quark interactions via exchanging the color charges. Lagrangian of the QCD contains of course the gauge field part, kinetic term of the field interacting part and mass.

$$\mathcal{L}_{QCD} = -\frac{1}{4}\sum_{a=1}^{8} G_a^{\mu\nu} G_{\mu\nu}^a + \sum_{j=1}^{n_f} \bar{q}_j \left( i\not{D} - m_j \right) q_j \tag{2}$$

Here we have summation of each gluon and on the right side for different flavors.

$$D = \partial_\mu + i g_s G_\mu \tag{3}$$

$g_s$ is the coupling constant for the strong interaction.

$$G_\mu = \sum_a \lambda^a G_\mu^a \tag{4}$$

where $G_\mu^a$ are gluon fields and $\lambda^a$ are the generators.

$$G_\mu^{a'} = G_\mu^a - \partial_\mu \alpha_a - g_s f_{ijk} \alpha_i G_\mu^j \tag{5}$$

The last term arises because we have non-abelian group and commutation relations are $[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k$

There are many experiments that give us clear evidence about existence of the quarks. But, free quarks are not observed and this is explained by the hypothesis of color confinement. Quarks interact by exchanging gluons that have color, because of that these are attractive interactions. Colour field is squeezed between the quarks and they are somewhat confined to a tube. The energy stored in the field is **proportional to the separation** of the quarks giving a term in the potential. So,total potential for quark and anti-quark can be proportional:

$$V_{q\bar{q}} \propto \left[\frac{\alpha_s(r)}{r} + \cdots + \kappa\, r\right] \tag{6}$$

Here $\alpha_s = g_s^2/4\pi$.

If we try to separate quark and anti-quark pair it will be energetically favourable to create additional pair and neutralize color. So, ultimately, we will have two jets of colorless hadrons. Consequently, we can say that this hypothesis about confinement gives us strong restrictions on the possible combinations of quarks and antiquarks. In this way we know that it is **impossible** to have $(q\bar{q})$ state [7]

## 2.4 Electroweak Theory

Force carrying particle in Electromagnetic interaction and in the weak interaction possess charge -1. This is the first implication that they might be somehow connected. Moreover, there are strong theoretical arguments that involve violation of quantum mechanical unitarity. For instance, $W^{\pm}$ can be produced by electron-positron annihilation, however exchange by neutrino and photon give a cross section that at high energies goes to infinity unless there exists additional gauge boson, neutral $Z$.

Now let us write the form of the SM Lagrangian for Electroweak part that consists of Yukawa, Higgs and gauge parts. For the last one we have kinetic terms for fermions and gauge fields:

$$\mathcal{L}_{gauge} = i\sum_{\alpha} \bar{f}_{\alpha L} \not{D} f_{\alpha L} + i\sum_{\alpha} \bar{f}_{\alpha R} \not{D} f_{\alpha R} - \sum_{a} \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \tag{7}$$

Here covariant derivative is defined as:

$$D_\mu = \partial_\mu + ig\frac{\tau^a}{2} W^a_\mu + ig' Q_Y B_\mu \tag{8}$$

$g'$ and $g$ are gauge couplings constants for $U(1)_Y$ and $SU(2)_L$. $\tau^a$ corresponds to the Pauli matrices for $SU(2)$ group. We cannot just write mass terms in SM Lagrangian because it violates gauge invariance, therefore we have Higgs part, that generates masses of the particles after Spontaneous Symmetry Breaking (SSB). The Goldstone theorem states that if we have continuous symmetry that is broken we get spin zero,massless particle, the Goldstone Boson [8].

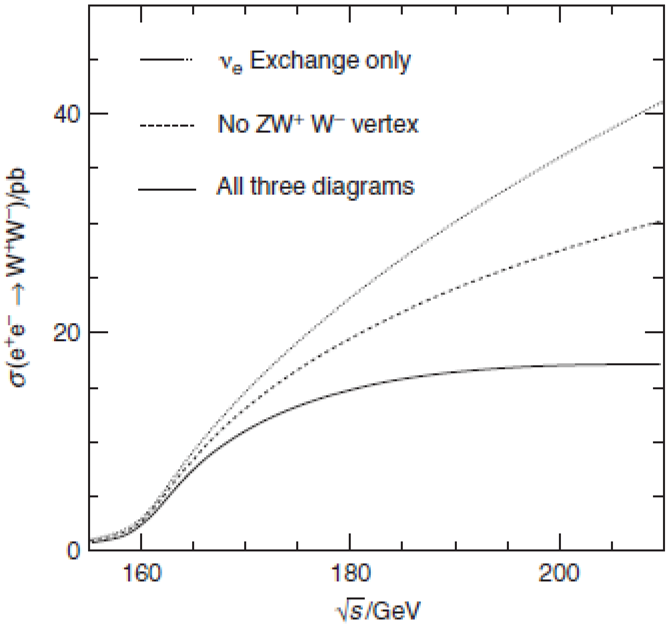



Figure 2: you can see the cross section for three cases: only neutrino exchange, neutrino and $\gamma$ exchange and all three exchange [7].

$$|\pi(\vec{p})\rangle = -\frac{2i}{F}\int d^3x e^{i\vec{p}\cdot\vec{x}} J_0(x)\,|\Omega\rangle[8]. \tag{9}$$

SSB occurs in a gauge theory with massless vector bosons and scalar fields, the Goldstone bosons transfer their degrees of freedom to the longitudinal mode of the vector fields and make them massive. This is the **Higgs Mechanism**. The Lagrangian has the form:

$$\mathcal{L}_{Higgs} = (D_\mu\Phi)^\dagger (D^\mu\Phi) - \mu^2\Phi^\dagger\Phi - \lambda\left(\Phi^\dagger\Phi\right)^2 \tag{10}$$

To generate masses, so to break the symmetry $\mu^2$ must be negative. So the middle part of the Higgs Lagrangian is not actually the mass term. Higgs potential has a set of minimum values that can be determined by:

$$\Phi^\dagger\Phi = \frac{v^2}{2} = \frac{-\mu^2}{2\lambda} \tag{11}$$

Now, if we choose one of the minimums we can write for the Higgs field as:

$$\Phi = \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} e^{i\frac{\pi(x)}{F}} \tag{12}$$

| | |
|---|---|
| $\sin^2\theta_W$ | 0.23126(5) |
| $M_W$ | $80.379 \pm 0.012$ GeV |
| $M_Z$ | $91.1876 \pm 0.0021$ GeV |
| $M_H$ | $125.10 \pm 0.14$ GeV |
| $v$ | $\sim 246$ GeV |

Table 1: List of EW theory parameters [9].

Here $H(x)$ is the physical Higgs boson and its excitation can change energy. $\pi(x)$ is the excitation on the same energy level, it's the Goldstone boson.

In unitary gauge we neglect oscillations that does not change the energy so:

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{13}$$

When we insert this field into the Lagrangian 3 bosons and Higgs field acquire masses while photon remains massless, so the mass part of the Lagrangian will have the form:

$$\mathcal{L}_{mass} = -M_H^2 H^2 + M_Z^2 Z_\mu^2 + M_W^2 W_\mu^\dagger W^\mu \tag{14}$$

Explicit calculation that we have omitted here gives us expressions for the masses that are: $M_H = \sqrt{s\lambda v^2}$ , $M_W = \frac{gv}{2}$ ,$M_Z = \frac{gv}{2cos(\theta_W)}$

Here $\theta_W$ is the weak mixing angle that is called Weinberg angle. Mixing happens between the neutral fields of the $U(1)_Y$ and $SU(2)_L$ gauge symmetries. This angle is connected to the gauge coupling constants in this way:

$$\tan\theta_W = \frac{g^{'}}{g} \tag{15}$$

Electric charge gets this form:

$$e = \frac{gg^{'}}{\sqrt{g^2 + g^{'2}}} = gsin\theta_W \tag{16}$$

Now let us see the Yukawa term in the Lagrangian, specifically the quark sector.

$$\mathcal{L}_{Yuk} = Y_{ij}^d \bar{Q}^i H d_R^j + Y_{ij}^u \bar{Q}^i \tilde{H}_R^j u_R^j \tag{17}$$

That explicitly is:

$$\mathcal{L}_{Yuk} = Y^d_{ij}\bar{Q}^i \begin{pmatrix} 0 \\ v \end{pmatrix} d^j_R + Y^u_{ij}\bar{Q}^i \begin{pmatrix} v \\ 0 \end{pmatrix} u^j_R \tag{18}$$

Doing further calculations give us the Lagrangian with the following form:

$$\mathcal{L}_{Yuk} = vY^d_{ij}\bar{d}^i_L d^j_R + vY^d_{ij}\bar{u}^i_L u^j_R \tag{19}$$

Which can be easily written in the matrix form:

$$\mathcal{L}_{Yuk} = v\left[\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R\right] \tag{20}$$

Now our aim is to diagonalize Yukawa matrices to get mass terms for quarks. Consequently, let us introduce diagonal matrices, such as: $M_u$ and $M_d$ and lets state that $Y_u Y_u^+ = U_u M_u^2 U_u^+$. From here we can write that $Y_u = U_u M_u K_u^+$. Now let us put this expression for Yukawa matrices into our Lagrangian.

$$\mathcal{L}_{Yuk} = v\left[\bar{d}_L U_d M_d K_d^+ d_R + \bar{u}_L U_u M_d K_u^+ u_R\right] \tag{21}$$

If we transform our fields in this way:

$$d_L \to U_d d_L \quad d_R \to K_d d_R u_L \to U_u u_L \quad u_R \to K_u u_R \tag{22}$$

We would eliminate extra terms in the Lagrangian.

$$\mathcal{L}_{Yuk} = v\left[\bar{d}_L M_d d_R + \bar{u}_L M_d u_R\right] \tag{23}$$

Now let us see the interaction term.

$$\mathcal{L}_{Yuk} = ... + W^+_\mu \bar{u}^i_L \gamma^\mu U_u^{+i} U_d^j d^j_L + W^-_\mu \bar{d}^i_L \gamma^\mu U_d^{+i} U_u^j d^j_L \tag{24}$$

we make denotion of the middle unitary terms $U_u^+ U_d = V_{CKM}$. This is well known Cabibbo-Kobayashi-Maskawa matrix, that embodies very interesting physical phenomena-quark mixing. It is a $3 \times 3$ matrix that in general is described by nine complex parameters. However, it is also unitary and this feature leaves us only nine independent parameters. It CKM were real we would get just orthogonal rotations with 3 parameters(rotations) however now we have other six additional degrees that can be complex phases of the form $e^{i\delta}$. This phases can be rotated in a way that from 6 there will be left only 1. Therefore, we can say that CKM has 4 parameters, 3 angles and 1 phase [7].

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \times \tag{25}$$

$$\times \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \times \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

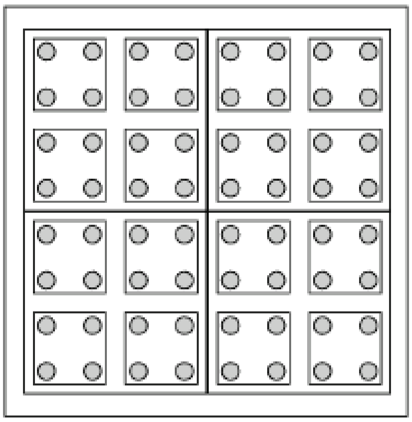
where $s_{ij} = sin(\phi_{ij})$ and $c_{ij} = cos(\phi_{ij})$
So, $V_{CKM}$ equals to:

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{26}$$

# 3 Renormalization Group

## 3.1 Simple example

We are now going to briefly discuss one of the most vital ideas in theoretical physics. As you know a change in scale is a scale transformation and our RG is connected to scale invariance and conformal invariance (symmetries where system appears same at all scale). Let us indroduce a Block Spin RG to better understand idea of this mathematical tool. If we consider 2D solid and we put atoms into a perfect imaginary squares we will get picture below:



Atoms interact among their nearest neighbours and system has temperature $T$. Hamiltonian of the system is $H(T, J)$, where $J$ characterizes the strength of the interaction. Now we divide system into solid blocks as it is in a picture and we introduce variables that describe not one by one interaction but average behaviour of the blocks. In the first approximation, we can say that total system hamiltonian will have nearly the same form $H(T', J')$. Consequently, if initially we had a lot of atoms and solving this problem was too hard now we made everything easier. We can continue this division into groups until there is only one very big block. This process is equivalent to finding the long range behaviour of the RG. There are two versions of the renormalization group used in quantum field theory. **The wilsonian renormalization group** tells us that with a UV cutoff $\Lambda$, physics at energies less than this coefficient is independent of precise value of it. Furthermore, changing the $\Lambda$ changes the couplings, however observables are the same. **The continuum renormalization group** tells us that observables do not depend on the renormalization conditions ( for instance on a scale $p_0$). This invariance holds when we renormalize theory and remove cutoff ($\Lambda = \infty$‘, ‘$d = 4$). We should mention also that in dimensional regularization the scales $p_0$ will be replaced by new undefined scale and our continuum RG comes from this scale independence. One should mention that these two are closely related but different. However, in both cases theory is independent of something. If we denote an observable by $\Theta$ we can say that $\frac{d}{d\Lambda}\Theta$‘ = ‘0,$\frac{d}{dp_0}\Theta$‘ = ‘0 or $\frac{d}{d\mu}\Theta$‘ = ‘0. When we solve these equations it will give us a trajectories and RG evolution refers to the flow along them. So what does the RG solve? well for example it can solve problem of **large logarithms**. If we calculate the vacuum polarization diagrams at 1-loop we end up with this:

$$V(p^2) = \frac{e_R^2}{p^2}\left(1 + \frac{e_R^2}{12\pi^2} ln\frac{p^2}{p_0^2}\right) \tag{27}$$

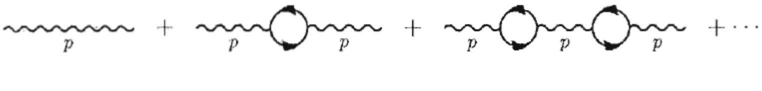
coefficient that logarithm has is or the order of $10^{-3}$ however there exists scales where $p^2 \gg p_0^2$. Okay, now let us solve our problem by RG that is large logarithms. We know that large logarithms are related to the physical scale $p^2$. At this scale potential were measured to an arbitrary scale $p_0^2$ and coupling was defined. The RGE then requires that our potential is independent of $p_0^2$.

$$p_0^2 \frac{d}{dp_0^2} V(p^2) = 0; \tag{28}$$

When we do all the calculations we get that:

$$e_{eff}^2(p^2) = \frac{e_R^2}{1 - \frac{e_R^2}{12\pi^2} ln\frac{p^2}{p_0^2}} \tag{29}$$

This is known as a running coupling. It includes contributions from all orders in perturbation theory.




We should specify that RG comes from the simple observation that there is nothing special about the renormalization point [8].

## 3.2 Gauge Couplings are Running

Let us now consider general aspects of the gauge coupling unifications. Here we just will use 1-loop level. The coupling constants $g_{1,2,3}$ of the standard gauge factors $SU(3) \times SU(2) \times U(1)$ at low energies are quite well known. Firstly we will write it as $\alpha$ that equals to $\alpha_i = \frac{g_i^2}{4\pi}$. We calculate it an Z-peak and they are the following [5]

$$\alpha_1^{-1}(M_Z) = 58.98 \pm 0.04;$$
$$\alpha_2^{-1}(M_Z) = 29.57 \pm 0.03;$$
$$\alpha_3^{-1}(M_Z) = 0.119 \pm 0.002;$$

From Renormalization Group Equations there are some vital parts that we use in our task. For example we will need to know from where this $B_t$ comes from that we are using so many times. Therefore, I will write everything down.

Yukawa constants are running too and we will see how. Let us just write 1-loop corrections that is suffice in our problem.

$$\frac{d}{dt} g_a = \frac{g_a^3}{16\pi^2} B_a^{(1)}; \tag{30}$$

Here $B_a^{(1)} = \left(\frac{33}{5}, 1, -3\right)$ for $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ respectively. Now the Yukawa coupling constant renormalization has a form:

$$\frac{d}{dt} Y_{u,d,e} = \frac{1}{16\pi^2} \beta^{(1)}_{Y_{u,d,e}} \tag{31}$$

now let us specify what is beta:

$$\begin{aligned}
\beta^{(1)}_{Y_u} &= Y_u 3Tr(Y_u Y_u^\dagger) + 3Y_u^\dagger Y_u + Y_d^\dagger Y_d - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \\
\beta^{(1)}_{Y_d} &= Y_d Tr(3Y_u Y_u^\dagger + Y_e Y_e^\dagger) + 3Y_d^\dagger Y_d + Y_u^\dagger Y_u - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \\
\beta^{(1)}_{Y_e} &= Y_e Tr(3Y_d Y_d^\dagger + Y_e Y_e^\dagger) + 3Y_e^\dagger Y_e - 3g_2^2 - \frac{9}{5} g_1^2
\end{aligned} \tag{32}$$

Of course, the $\beta$ functions are $3 \times 3$ matrices in family space [15].

# 4 Fritzsch Mass Matrices

Firstly, let us state that $\Psi^U = (u, c, t)$, $\Psi^D = (u, c, t)$ and $\Psi^E = (u, c, t)$. So, mass matrices for these fields are denoted by $M^U$,$M^D$ and $M^E$. Fritzsch with his ansatz introduced mass matrices that have the form [13]:

$$\bar{M} = \begin{pmatrix} 0 & Ae^{i\alpha} & 0 \\ \tilde{A}e^{i\tilde{\alpha}} & 0 & Be^{i\beta} \\ 0 & \bar{B}e^{i\tilde{\beta}} & Ce^{i\gamma} \end{pmatrix} \tag{33}$$

these coefficients have the hierarchy $C \gg B, \tilde{B} \gg A, \tilde{A}$. The mass matrices of type U,D and E are of this form. It is convenient to write $\tilde{M}$ in a diverse form $\tilde{M} = P_1 M P_2$. Here $P_j$ is a diagonal matrix with phases and it will remove all phase from our $\tilde{M}$ mass matrix. If we assume that $P_j = diag(e^{i\alpha_j}, e^{i\beta_j}, e^{i\gamma_j})$ (j=1,2), then we can derive connection:

$$\begin{aligned} \alpha &= \beta_2 + \alpha_1, \\ \tilde{\alpha} &= \beta_1 + \alpha_2, \\ \beta &= \gamma_2 + \beta_1, \\ \tilde{\beta} &= \gamma_1 + \beta_2, \\ \gamma &= \gamma_1 + \gamma_2, \end{aligned} \tag{34}$$

If we make more simplification and assume that $\alpha = \tilde{\alpha}$, $\beta = \tilde{\beta}$, $A = \tilde{A}$ and $P_1 = P_2$.

So, at the end we get, very simplified matrix M:

$$\tilde{M} = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} \tag{35}$$

The texture of Fritzsch naturally comes in the context of SU(5) however, theoretical approach to these types of matrices have some problems when comparing with the experimental ones. It was nearly impossible to match all the parameters simultaneously, until now! Because we have succeeded. Fritzsch tried to use symmetric matrices, with less parameters, however, he failed and could not fulfil experimental expectations [14],[16]. The reason was very simple, he was just taking random matrices and using them, there were no theoretical model from which these textures would be realized. However, in our case everything comes from the model and it is physical. Okay we stopped on the symmetric matrix that has free parameters that must be fixed somehow by known masses. It must be also emphasized that these procedures are used in the future to get way difficult equations, therefore these subsections are important to understand every detail in the thesis. To fix free parameters: A, B and we want to make diagonalization of these matrices to get masses. We say initially that the eigenvalues are $(M_1, -M_2, M_3)$. By taking trace of matrix M, its determinant and trace of its square ($tr(M)$, $det(M)$ and $tr(M^2)$) we can get these relations between coefficients of matrix M (A,B and C) and eigenvalues [17].

$$\begin{aligned} C &= M_1 - M_2 + M_3, \\ A^2 C &= M_1 M_2 M_3 \\ 2(A^2 + B^2) + C^2 &= M^1 + M^2 + M^3 \end{aligned} \tag{36}$$

From here it is easy to calculate exact values of coefficients:

$$\begin{aligned} C &= M_1 - M_2 + M_3, \\ A &= \sqrt{\frac{M_1 M_2 M_3}{M_1 - M_2 + M_3}} \\ B &= \frac{(M_3 - M_2)(M_3 + M_1)(M_2 - M_1)}{M_1 - M_2 + M_3} \end{aligned} \tag{37}$$

Now, if we use hierarchy of the masses we can easily say that $C \approx M_3$, $A \approx \sqrt{M_1 M_2}$ and $B \approx \sqrt{M_2 M_3}$ finding coefficients enable us to get matrices that make diagonalization of our Fritzsch matrix. Therefore our CKM matrix will be

$$V_{CKM} = O_u^+ P_u^* P_d O_d \tag{38}$$

CKM was already defined in the previous sections therefore we will not redefine it again.

# 5 Beyond Standard Model

Understanding flavour dynamics and the related origin of quark and lepton masses and mixings are one of the most important aims in particle physics. In this context, weak decays of hadrons, and in particular the CP violating and rare decay processes, play crucial role as they are sensitive to short distance phenomena. Therefore, determination of the CKM that parametrizes the weak charged current interactions of quarks is currently a central theme in particle physics.

## 5.1 SUSY

Generally, Supersymmetry is a model that proposes connection between bosons and fermions. In SUSY each particle from one group would have an associated particle in the other and they are called superpartners. However, it is **not like an** antiparticle because superpartners differ by a half-integer spin. There are many Motivations for SUSY. One of them is Gauge coupling unification. You will see on a picture below, that Just GUT is not enough for uniting coupling constants at $10^16$ $GeV$ scale. They come very close, however it is the SUSY that make them to go in one point. Therefore, we can say that without SUSY GUT is not complete. Superpartners are called different names like Higgsinos, Gluinos and etc. Now what about model. Firstly, let us just understand what is this SUSY exactly, what type of symmetry it exerts on a system or, what kind of symmetry, it is. To understand better what is SUSY let us introduce a simple free model that consists of one massive fermion of mass $m$, denoted by $\psi(x)$ and two complex scalars of the same mass $m$ that will be denoted by $\phi_+(x)$ and $\phi_-(x)$. Lagrangian for this simple model will be:

$$\mathcal{L} = \partial^\mu \phi_+^* \partial_\mu \phi_+^* - m^2 \left|\phi_+\right|^2 + \partial^\mu \phi_-^* \partial_\mu \phi_-^* - m^2 \left|\phi_-\right|^2 + \bar{\psi}(\imath\gamma^\nu \partial_{nu} - m)\phi \quad (39)$$

This theory has symmetries like: spacetime, translation, rotations and boosts. We here have spinor that has left and right handed parts. However, we can write right-handed spinor in terms of a left-handed.

$$\psi_R = \imath\sigma_2 \psi_L^*; \quad (40)$$

Now let us introduce two left-handed spinors $\psi_+$ and $\psi_-$ and define our old spinor with them;

$$\psi_L = \psi_+, \qquad \psi_R = \imath\sigma_2 \psi_-^*; \quad (41)$$

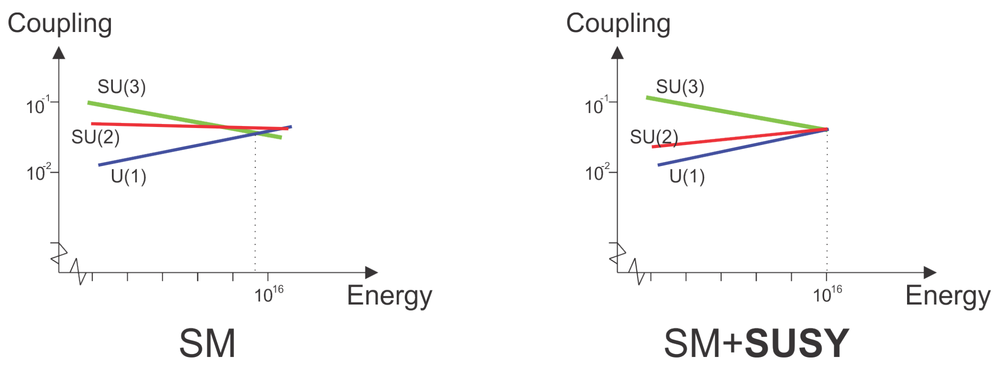
Now let us put everything into our main lagrangian:

$$\begin{aligned} \mathcal{L} = \partial^\mu \phi_+^* \partial_\mu \phi_+^* - m^2 \left|\phi_+\right|^2 + \partial^\mu \phi_-^* \partial_\mu \phi_-^* - m^2 \left|\phi_-\right|^2 + \\ \psi_+^\dagger \imath\bar{\sigma}^\mu \partial_\mu \psi_+ + \psi_-^\dagger \imath\bar{\sigma}^\mu \partial_\mu \psi_- - m(-\psi_+^T \imath\sigma_2 \psi_- + h.c) \end{aligned} \quad (42)$$

Now what we do is called "Extension of the Symmetry". We take two-component left spinor and make following transformation:

$$\delta_\xi \psi_+ = \sqrt{2}\imath\sigma^\mu\epsilon\xi^*\partial_\mu\phi_+ \qquad (43)$$
$$\delta_\xi \phi_+ = \sqrt{2}\xi^T\epsilon\psi_+$$

So, it is evident that such symmetry transformation takes fermion into a boson and vice versa. This is called a SUSY. It must be noted that masses should be same otherwise theory will not be supersymmetric




To incorporate **SUSY** into our SM requires doubling the number of particles. When we add new particles we have a lot of other interactions. Now let us consider the most general SUSY-GUT model then lowering energy we will get Minimal Supersymmetric model consistent with the SM, which is called Minimal Supersymmetric SM (MSSM). This is a minimal phenomenologically viable extension of the SM, that adds the least number of extra elements. Consequently, In the MSSM, each of our fundamental particle has its superpartner with different spin. Now on the table below let us specify them (superpartners are identified by tilde):

| Names | | Spin 0 | Spin 1/2 | $I$ | $I_3$ | $Y$ | $Q$ |
|---|---|---|---|---|---|---|---|
| SLeptons, Leptons | L | $(\tilde{\nu}_{lL}, \tilde{l}_L)$ | $(\nu_{lL}, l_L)$ | 1/2 | $(+1/2, -1/2)$ | $-1$ | $(0, -1)$ |
| | $\bar{l}$ | $\tilde{l}_R$ | $l_R$ | 0 | 0 | $-2$ | $-1$ |
| SQuarks, Quarks | Q | $(\tilde{u}_L, \tilde{d}_L)$ | $(u_L, d_L)$ | 1/2 | $(+1/2, -1/2)$ | 1/3 | $(2/3, -1/3)$ |
| | $\bar{u}$ | $\tilde{u}_R$ | $u_R$ | 0 | 0 | 4/3 | 2/3 |
| | $\bar{d}$ | $\tilde{d}_R$ | $d_R$ | 0 | 0 | $-2/3$ | $-1/3$ |

Table 2: Fermions content of MSSM and its quantum numbers with respect to the EW gauge group. Here, $u = u,c,t$ for up quark and $d = d,s,b$ for down quark while $l = e,\ \mu,\ \tau$

| Names | Spin 1/2 | Spin 1 | $I$ | $I_3$ | $Y$ | $Q$ |
|---|---|---|---|---|---|---|
| Higgsinos, Higgs | $(\tilde{H}_u^+, \tilde{H}_u^0)$ | $(H_u^+, H_u^0)$ | 1/2 | (+1/2, −1/2) | +1/2 | (+1, 0) |
| | $(\tilde{H}_d^0, \tilde{H}_d^-)$ | $(H_d^0, H_d^-)$ | 1/2 | (+1/2, −1/2) | −1/2 | (0, −1) |
| Gluino, Gluons | $\tilde{g}$ | $g$ | 0 | 0 | 0 | 0 |
| Winos, W Bosons | $(\tilde{W}^\pm, \tilde{W}^0)$ | $(W^\pm, W^0)$ | 1 | (±1, 0) | 0 | (±1, 0) |
| Bino, B Boson | $\tilde{B}^0$ | $B^0$ | 0 | 0 | 0 | 0 |

Table 3: Bosons content of MSSM and its quantum numbers with respect to the EW gauge group.

Before starting let me define two important terms one is called F and another D. For each vector multiplet $(\lambda^a, A_\mu^a)$ we have:

$$D^a = -g\phi_i^\dagger T^a \phi_a \tag{44}$$

To this all the charged scalars contribute. For each chiral multiplet $(\phi_i, \psi_i)$ we have:

$$F*_i = -\frac{\partial \mathcal{W}}{\partial \phi_i} \tag{45}$$

The scalar potential is:

$$V = F*_i F_i + \frac{1}{2} D^a D^a \tag{46}$$

With this language we can now write everything. Okay, let us start. We are considering SUSY-GUT based symmetry within a framework of N=1 Supersymmetry. That means that such a theory must contain vector superfields V in adjoint representation of our symmetry. We set chiral superfields $\Phi$ in a different representations of our symmetry. Let us write a generic renormalizable Lagrangian

$$\mathcal{L}_{SUSY} = \int d^2\theta d^2\bar{\theta}\Phi^\dagger e^V \Phi + \left[\int d^2\mathcal{W}\mathcal{W} + \int d^2\theta W(\Phi) + h.c\right]$$

Here, first and second term give us canonically normalized kinetic terms and gauge superfields with their gauge interactions. Third therm describes mass and intecation terms between the fermionic and scalar components of chiral superfields $\Phi$, via the superpotential $\mathcal{W}(\Phi)$.

To break the symmetry and introduce spontaneous symmetry breaking in the model we add "soft" terms that has nearly the same forms and is presented in a same way as the first Lagrangian:

$$\mathcal{L}_{SSB} = \int d^2\theta d^2\bar{\theta}\rho\Phi^\dagger e^V \Phi + \left[\int d^2\theta\eta\mathcal{W}\mathcal{W} + \int d^2\theta\eta\mathcal{W}'(\Phi) + h.c\right]$$

So, what we did is just used auxiliary superfields with non-zero F and D-terms ,where $\eta = M_F\theta^2$ and $\rho = M_D^2\theta^2\bar{\theta}^2$ this $M_F$ and $M_D$ can be different. First one determines the size of gaugino mass terms.

The chiral superfields can be divided into Higgs and fermion superfields distinguished by matter parity $Z_2$, under which the fermion superfields change the sign while the Higgs superfields are invariant. In this way we have this form:

$$\mathcal{W} = \mathcal{W}_{Higgs} + \mathcal{W}_{Yukawa}$$

This Higgs part is responsible for the VEVs breaking both the gauge symmetry that we introduced to MSSM and then to SM.

Now what is the MSSM, it is the low energy limit of SUSY-GUT. In this low energy mode, superpotential has the form:

$$\mathcal{W}_{MSSM} = Y^u_{ij}Q_iu^c_jH_u + Y^d_{ij}Q_id^c_jH_d + Y^e_{ij}e^c_iL_jH_d + \mu H_uH_d \tag{47}$$

This part contains chiral superfields corresponding to three families of quarks and leptons and two Higgses $H_u$ and $H_d$. The SSB F-terms repeat the structure of $\mathcal{W}_{MSSM}$ and contain also the soft Majorana masses of gauginos:

$$\mathcal{L}_F = A^u_{ij}\tilde{Q}_i\tilde{u}^c_jH_u + A^d_{ij}\tilde{Q}_i\tilde{d}^c_jH_d + A^e_{ij}\tilde{e}^c_i\tilde{L}_jH_d + \mu H_uH_d + \tilde{m}^a_G\lambda_a\lambda_a \tag{48}$$

Soft masses of all scalars including the Higgses are given by D-terms:

$$\begin{aligned}\mathcal{L}_D = \tilde{m}^2_{Qij}\tilde{Q}^\dagger_i\tilde{Q}_j + \tilde{m}^2_{uij}\tilde{u}^{\dagger c}_i\tilde{u}^c_j + \tilde{m}^2_{dij}\tilde{d}^{\dagger c}_i\tilde{d}^c_j + \\ +\tilde{m}^2_{Lij}\tilde{L}^{\dagger c}_i\tilde{L}^c_j + \tilde{m}^2_{eij}\tilde{e}^{\dagger c}_i\tilde{e}^c_j + \tilde{M}^2_uH^*_uH_u + \tilde{M}^2_dH^*_dH_d\end{aligned} \tag{49}$$

Let us comment on their mass matrix:

$$\mathcal{M}^2 = \begin{pmatrix} \tilde{M}^2_u + \mu^2 & \mu B_\mu \\ \mu B_\mu & \tilde{M}^2_d + \mu^2 \end{pmatrix}$$

As you can see the mass matrix contains mass terms $\tilde{M}_u^2$ and $\tilde{M}_d^2$ that are D-terms of spontanious symmetry breaking and $B_\mu$ are F-terms. $\mu$ is a supersymmetric term.

Okay we have described sufficiently good model, however, there are some other terms that could be added. For example, some quark to lepton direct transition that violates baryon-lepton number and etc. Therefore, we must introduce something that will fix and tell me that **NO** you cannot add terms like that. Therefore, we introduce new discrete symmetry that is called **R-parity**. Sometimes it is called matter parity. Then we can postulate B and L conservation. Moreover, $P_R = +1$ for all SM particles and $P_R = -1$ for all sparticles.

$$P_R = (-1)^{3(B-L)+2s} \tag{50}$$

Conservation of **R-parity** states the difference between our particles and their superpartners. Also, in every interaction vertex we have conservation of **R-parity** One subtle remark is that last term in the Lagrangian $\mu H_u H_d$ can become huge therefore, we might have hierarchy problems here. For this reason we add terms that somehow, diminish this huge part of the Lagrangian. We have not observed superpartners yet, therefore some realistic phenomenological model must contain SUSY breaking terms. So, we add **soft** term to MSSM to maintain hierarchy berween EW scale and Planck scale.

$$\mathcal{L} = \mathcal{L}_{MSSM} + \mathcal{L}_{Soft} \tag{51}$$

The main difference between SSB here and EW symmetry breaking is that we have two Higgs bosons that have two VEVs $v_u$ and $v_d$. This VEVs of course have restriction by SM: $v_u^2 + v_d^2 = v^2 \approx (174GeV)^2$. From this equation we can easily parametrize our VEVs:

$$v_u = \langle H_u^0 \rangle = v \sin\beta \qquad v_d = \langle H_d^0 \rangle = v \cos\beta \qquad \tan\beta = \frac{v_u}{v_d} \tag{52}$$

SSB gives us the process in which three of the eight degrees of freedom of the Higgs fields become the longitudinal modes of the physical $Z^0$ and $W^\pm$. From the equations above it is clear that in MSSM, masses and mixing angles for quarks and leptons depend not only on Yukawa coupling but also on parameter $\beta$.

## 5.2 GUT

### 5.2.1 SU(5)

A **Grand Unified Theory** is just a guess that there exist much more symmetries in the nature than we have in the SM and this symmetry depends on the energy. So, SM is low energy limit of GUT. The SM has been tested countless times and as we already mentioned it is incomplete. However, we cannot prove experimentally that GUT fixes something but we cannot see otherwise too. GUT is a model that states that at high energy, our three gauge interactions of the SM (electromagnetic weak and strong) are merged into a larger gauge symmetry, one single force with one single parameter. If we have realization of GUT in our nature, then it is logical to think that there was a epoch of grand unification in the early universe when fundamental forces were not distinct. So, currently we are leaving in a broken phase in which $SU(3)_C \times U(1)_Q$ is invariant to us and the low-energy phenomena are governed by the electrodynamics and strong interactions. The first implication of the GUT is that at some scale $M_U \gg M_W$ the relevant symmetry is $G$ and $g_1$ , $g_2$ and $g_3$ coupling constants of $SU(3)_C \times SU(2)_L \times U(1)_Y$ merge into a single gauge coupling $g_U$ The most important "hints" in favour of GUT is the fact that the running within the SM shows an approximate convergence of the gauge couplings around $10^{15}$ GeV. In our problem we consider Georgi-Glashow Model therefore let us talk a little bit about it.

As you know in SM we have groups of rank four:$SU(3)$ has rank 2, SU(2) rank 1 and $U(1)$ rank 1. Therefore, If we require that rank of the group that contains our SM must be the same then the only simple group with complex representations which can contain our SM is $SU(5)$. This groups' fundamental representation is a 5-dimensional vector $5_i$ and embedding is in a way that upper part goes for three components of $SU(3)$ and down for $SU(2)_L$.

$$5 = (3, 1) \oplus (1, 2) \tag{53}$$

Second representation of $SU(5)$ is 10. Ten is a little bit complex than 5.

$$10 = (5 \otimes 5)_A = (\bar{3}, 1, -\frac{2}{3}) \otimes (\bar{3}, 2, +\frac{1}{6}) \otimes (1, 1, +1) \tag{54}$$

To embed our SM particles in these representations we firstly must specify them:

$$q \sim \left(3, 2, +\frac{1}{6}\right), \quad l \sim \left(1, 2, -\frac{1}{2}\right),$$
$$u^c \sim \left(\bar{3}, 1, -\frac{2}{3}\right), \quad d^c \sim \left(\bar{3}, 1, +\frac{1}{3}\right), \quad e^c \sim (1, 1, +1) \tag{55}$$

Therefore, our particles will be embedded into $\bar{5} \oplus 10$. Also it must be mentioned that hypercharge is a traceless generator of $SU(5)$ and that is how the third component (Hypercharge)is taken. Adjoint representation of $SU(5)$ is 24 dimensional,traceless.

$$\Sigma = \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & b \end{pmatrix} \tag{56}$$

It has this form, When we insert in the process the most important thing that it does is "telling" to down-quarks and leptons that they are different. It gives them different coefficients and weights.

Yukawa sector in $SU(5)$ is quite interesting. it consists of 2 parts (and of course hermition conjugate):

$$\mathcal{L}_Y = \bar{5} \cdot Y \cdot 10\bar{5}_H + \frac{1}{8} 10 \cdot Y \cdot 10 \cdot 5_H + h.c \tag{57}$$

It is better not to write all the indices explicitly because otherwise it is impossible to see formulas, but we should say that we have family and Lorentz indices. First part on the left gives us down-quarks and leptons and the right one just up part. It is a subtle point because as down-quarks and leptons are written by the same operator they have the same coupling constants, however their Yukawa mass matrices are Transposed.

#### 5.2.2 SO(10)

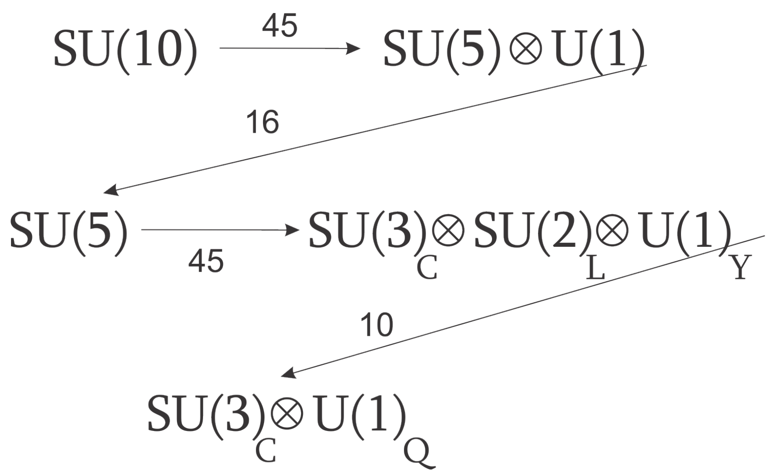
The next and the most interesting for us simple Lie group containing our Standard Model is $SO(10)$:

$$SO(10) \quad \supset \quad SU(5) \quad \supset \quad SU(3) \otimes SU(2) \otimes U(1)$$

Here unification of matter is way complete than it was in $SU(5)$, because here we do not have two representations in which we embed our SM particles-**we have only one** spinorial representation that is called 16. The Boson matrix for it is found by taking $15 \times 15$ matrix from the $10 + \bar{5}$ representation and adding 1 raw and column for right-handed neutrino. Now let us talk about group itself. This group is orthogonal group of rotations in ten dimensional vector space. The matrices of our group can be written by 45 imaginary generators:

$$O = exp\left(\frac{1}{2}\epsilon_{ij}T_{ij}\right)$$

$SO(10)$ has two main "channels" to decompose ,just to mention, one of them is into $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ that is called Pati-Salam Model the other is what we want $SU(5) \otimes U(1)_X$. In the publication by R.Slansky there are brilliantly discussed how these decompositions happen [18]. We will shortly show diagram: and also must be seen hypercharges of decoupled




groups because we will see that hypercharges are connected to the Mass, consequently, knowing it is important.

$$16 = 1(-5) + \bar{5}(3) + 10(-1)$$
$$45 = 1(0) + \bar{1}0(-4) + 10(4) + 24(0)$$

Furthermore, tensor products are vital [18]:

$$10 \times 10 = 1_s + 45_a + 54s;$$
$$\bar{1}6 \times 10 = 16 + 144$$
$$16 \times 16 = 10_s + 120_a + 126_s$$
$$\bar{1}6 \times 16 = 1 + 45 + 210$$

Now about processes in $SO(10)$. As we have already mentioned theory here is better unified, the reason is adjoint spinorial representation 16 that contains everything, even Right-handed antineutrino. couplings are written as:

$$16 \cdot 16 \cdot 10_s \tag{58}$$

where this $10_s$ is for the Higgs part. Here everything has the same coupling constant.

Now let us more discuss GUT with a "different angle too". The basic tool for exploring GUT is the renormalization. Group evolution of the gauge couplings as a function of energy scale can be given by:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha(M_{th})} + \frac{b_i}{2\pi} \log\left(\frac{M_{th}}{\mu}\right) \tag{59}$$

where $\alpha_i = g_i^2/4\pi$ and the i=1,2,3 indicate which coupling constant we are considering they are linked to the SM coupling constant by the relations: $\sqrt{3/5}\, g_1 = g'$ , $g_2 = g$ and $g_3 = g_s$.

$M_{th}$ is a threshold where a new set of particles enter into the loop diagrams. It corresponds to the mass of superpartners to $M_{SUSY}$. The $b_i$ coefficients also depend on the number of particles in the loops:

$$\begin{aligned} b_1 &= -\left(\frac{4}{3}N_F + \frac{1}{10}N_H\right) \\ b_2 &= \left(\frac{22}{3} - \frac{4}{3}N_F - \frac{1}{6}N_H\right) \\ b_3 &= \left(11 - \frac{4}{3}N_F\right) \end{aligned} \tag{60}$$

Here $N_F$ is the number of families running into loops and N_H is the number of Higgs doublets. When we reach to SUSY our particle content will be enriched and our coefficients will gain some extra contributions:

$$\begin{aligned} \Delta b_1 &= -\left(-2 - \frac{2}{3}N_F\right) \\ \Delta b_2 &= \left(-\frac{4}{3} - \frac{2}{3}N_F - \frac{1}{3}N_H\right) \\ \Delta b_3 &= \left(-\frac{2}{3}N_F - \frac{1}{5}N_H\right) \end{aligned} \tag{61}$$

As we will see in the next section for SUSY GUT $N_H = 2$. With this modifications three gauge couplings meet on a single energy scale:

$$M_G = 2.19^{+0.44}_{-0.37} \times 10^{16}\, GeV \tag{62}$$

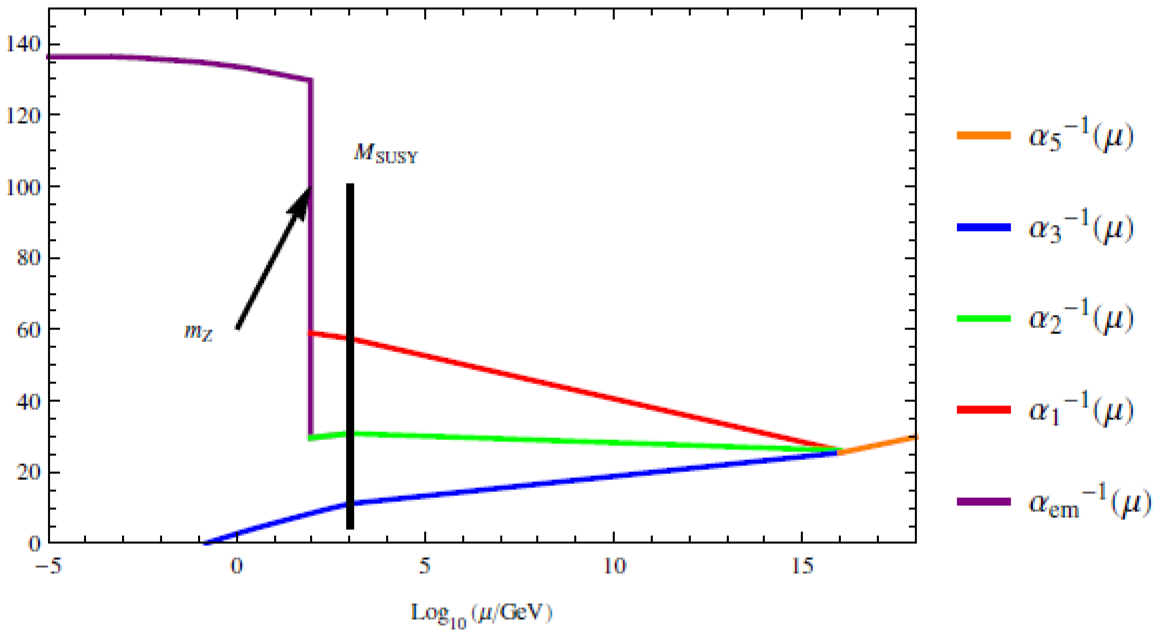



Figure 3: Running of the gauge coupling constants in the SU(5) SUSY + GUT scenario. SUSY and EW breaking scale are underlined.

# 6 The Model

Now let us discuss theoretical framework for our model. Here it is considered the supersymmetric $SU(5)$ model with addition of $SU(3)_H$ symmetry among the three families of quarks and leptons. With this we try to depict processes that might explain mass Hierarchy between families. As we know, standard $SU(5)$ fundamental representation is 5. Introducing new symmetry tells us that three fermion familie are unified in chiral superfields:

$$\bar{5} = (d^c, l)_i \sim (\bar{5}, 3) \quad 10_i = (u^c, q, e^c)_i \sim (10, 3). \tag{63}$$

These, $i$ indices come from $SU(3)$. So, now what about Higgs? Higgs superfields that are 5 and $\bar{5}$ are singlets of $SU(3)_H$

$$H \sim (5, 1), \quad \bar{H} \sim (\bar{5}, 1) \tag{64}$$

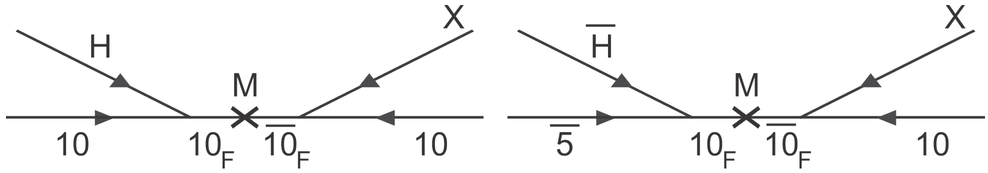
Of course, we assume that the theory is invariant under R-parity. Let us talk more about The Higgs sector. For our SU(5) it spans over the reducible $5_H \oplus 24_H$. These two fields are minimally needed in order to break SU(5). However,in our model, we have also new symmetry,therefore it forbids standard coupling with Higgses. The reason is that fermion bilinears transform as $3 \times 3 = \bar{3} + 6$ therefore, fermion masses can be introduced by higher order operators involving "horizontal" Higgs superfields $\chi_{\alpha\beta}$. This field will help us to break $SU(3)_H$ till the $SU(5)$. After that other two Higgs fields will carry on. The $\chi$ fields transform under $SU(3)_H$ either as sextets or as antitriplets [6]. Let us specify the general structure of the VEV.

$$U = \begin{pmatrix} A_{11} & A_{12} + B_{12} & A_{13} + B_{13} \\ A_{12} - B_{12} & A_{22} & A_{23} + B_{23} \\ A_{13} - B_{13} & A_{23} - B_{23} & A_{33} \end{pmatrix} \tag{65}$$

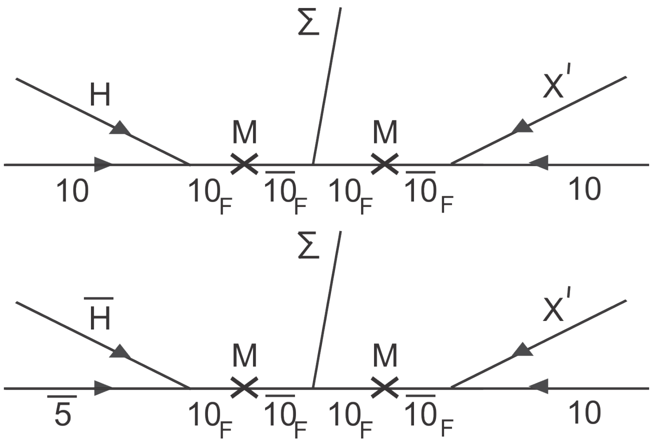
Here A and B come from the horizontal Higgs fields from the sextets and antitriplets respectively. However, we make a simplification and say that they both have a third direction. In this way in the sextet we will have only one (3,3) term and in antitriplet (1,3). In this way we only have (12),(21) and (33)element in $\chi$, that has Fritzsch-like matrix form and we know to to treat with it from previous sections. Now let us see the form of operators in our model.

$$O_u = \frac{\chi^{ij}}{M} 10_i 10_j H, \quad O = \frac{\chi^{ij}}{M} \bar{5}_i 10_j \bar{H} \tag{66}$$

I already said previously but it is very important part so let me strain that down-quarks and leptons are united in one operator, therefore there is one coupling constant for them and after breaking $SU(3)_H$ particles do not "Know" about it, therefore we should add something to the theory that will differentiate between the leptons and down-quarks. So what about $M$. M is just some large scale. It is well known that the effective operators (66) naturally arise from a renormalizable theory after integrating some heavy degrees of freedom [19]. Therefore, this flavour scale can be connected to the mass scale. It is convenient to say that this mass scale is set by large VEV of some Higgs $\Omega$, and $\langle\Omega\rangle \sim M$ We will show everything explicitly with diagrams and etc. It can be said that there are quark-like particles that can be represented by $\bar{5}_F \oplus 10_F$. These heavy particles take place into the several processes like:



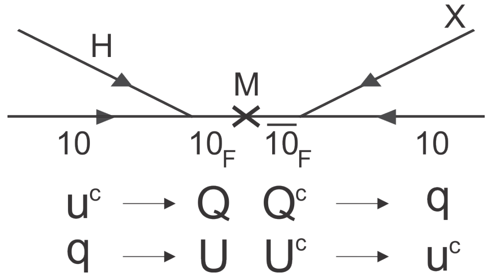

As you can see, we have here two Higgs fields, one from 5 representation and another for $SU(3)_H$ breaking. Additionally, we can change process and add one 24 representation Higgs field in the middle.




You might Ask why. Why do we need introduction of new, additional Higgs field. The reason is very clear. As we already emphasized downs and leptons are united, therefore these adjoint 24 will introduce Clebsch-Gordan coefficiets and differentiate between downs and leptons. After introducing new $\Sigma$ you can see that instead of old $\chi$ we have used $\chi'$ this is because we say that during these processes this sextet and triplet can be rotated by some angle $\alpha$.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2B & -scB \\ 0 & -scB & c^2B \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ A \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ As \\ Ac \end{pmatrix} \tag{67}$$

Now, we have to write operators explicitly count all combinatoric terms and write all Yukawa terms to get Yukawa matrices at the end. Let us now concentrate on the picture below where we have specified all terms of combinations in the process.




We can now construct a matrix that will have a form:

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ |
|---|---|---|---|
| $q$ | 0 | $\chi$ | $gH$ |
| $Q$ | $gH$ | $M$ | 0 |
| $U$ | $\chi$ | 0 | $M$ |

To continue, $M$ is big, consequently, it is normal to diagonalize this matrix like that:

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ |
|---|---|---|---|
| $q$ | $\frac{2gH\chi}{M}$ | 0 | 0 |
| $Q$ | 0 | $M$ | 0 |
| $U$ | 0 | 0 | $M$ |

So in this way we have an operator:

$$\frac{2g\chi_{\alpha\beta}}{M}\epsilon^{ijklm}10_{ij}10_{kl}H_m \tag{68}$$

Consequently, we can use same steps to construct operators for next order graphs where we have three Higgs fields.

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ | $Q^c_{\circledcirc}$ | $U^c_{\circledcirc}$ |
|---|---|---|---|---|---|
| $q$ | 0 | $\chi$ | $gH$ | $\chi'$ | 0 |
| $Q$ | $gH$ | $M$ | 0 | 0 | 0 |
| $U$ | $\chi$ | 0 | $M$ | 0 | $g'\Sigma$ |
| $Q^c_{\circledcirc}$ | 0 | $g'\Sigma$ | 0 | $M'$ | 0 |
| $U^c_{\circledcirc}$ | $\chi'$ | 0 | 0 | 0 | $M'$ |

At the end of the day when we sum up all the diagrams for the processes we get:

$$\begin{aligned} &\frac{2g_1\chi}{M}\bar{5}^i 10_{ij}\bar{H}^j + \frac{2g\chi}{M}\epsilon^{ijklm}10_{ij}10_{kl}H_m + \\ &+\frac{g_1 g_1'\chi'}{MM'}\Sigma_p^i \bar{5}^j 10_{ij}\bar{H}^p + \frac{g_1 g_1'\chi'}{MM'}\Sigma_p^i \bar{5}^p 10_{ij}\bar{H}^j + \\ +&\frac{gg'\chi'}{MM'}\epsilon^{pjklm}\Sigma_p^i 10_{ij}10_{kl}\bar{H}^p + \frac{gg'\chi'}{MM'}\epsilon^{pijkl}\Sigma_p^m 10_{ij}10_{kl}\bar{H}^m \end{aligned} \tag{69}$$

Here $\Sigma$ is an adjoint representation that has all elements 0 except diagonal ones. Moreover, trace is also 0.

$$\Sigma = \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & b \end{pmatrix} \tag{70}$$

When we sum up all the indices at the end of the day we will get Yukawa matrices for up, down and lepton parts, that will depend on A-B parameters (will be specified soon) on angle (6 and $3^*$ rotation) and a-b.

## 6.1 Preparation to test the Model

Yukawa coupling constants connect to the physical fermion masses, however masses are running (their value depend on the energy on which we are measuring it). Therefore, connection is through the renormalization group equations (RGE).

$tan\beta = v_2/v_1$ [12].

$$\begin{aligned} m_u = Y_u R_u \eta_u B_t^3 v_2,\ m_d = Y_d R_d \eta_d v_1,\ m_e = Y_e R_e v_1 \\ m_c = Y_c R_u \eta_c B_t^3 v_2,\ m_s = Y_s R_d \eta_s v_1,\ m_\mu = Y_\mu R_e v_1 \\ m_t = Y_t R_u B_t^6 v_2,\ m_b = Y_b R_d \eta_b B_t v_1,\ m_\tau = Y_\tau R_e v_1 \end{aligned} \tag{71}$$

Here the factors $R_u$,$R_d$ and $R_e$ account for the gauge-coupling induced running from $10^{16}$ to the SUSY breaking scale $M_s \sim M_t$. $\eta$ factors encapsulate the $QCD + QED$ running from $M_s$ to $m_f$ for $f = b, c$. It is said that $\eta = 1GeV$ for the light quarks ($f = u, d, s$). For $\alpha_s(M_Z) = 0.118 \pm 0.005$ we have:

$$\begin{aligned} R_u = 3.33 \pm 0.07, \quad R_d = 3.25 \pm 0.07, \quad R_e = 1.49 \\ \eta_b = 1.52 \pm 0.04, \quad \eta_c = 2.02^{+0.16}_{-0.13} \pm 0.07, \quad \eta_{u,d,s} = 2.33^{+0.29}_{-0.21} \end{aligned} \tag{72}$$

The factor $B_t$ includes the running induced by the large top quark Yukawa constant ($Y_t \sim 1$):

$$B_t = exp\left[-\frac{1}{16\pi^2}\int_{lnM_S}^{lnM_G} Y_t^2(\mu)d(ln\mu)\right]; \tag{73}$$

$B_t$ as a function of the GUT scale value is shown below on a picture: We

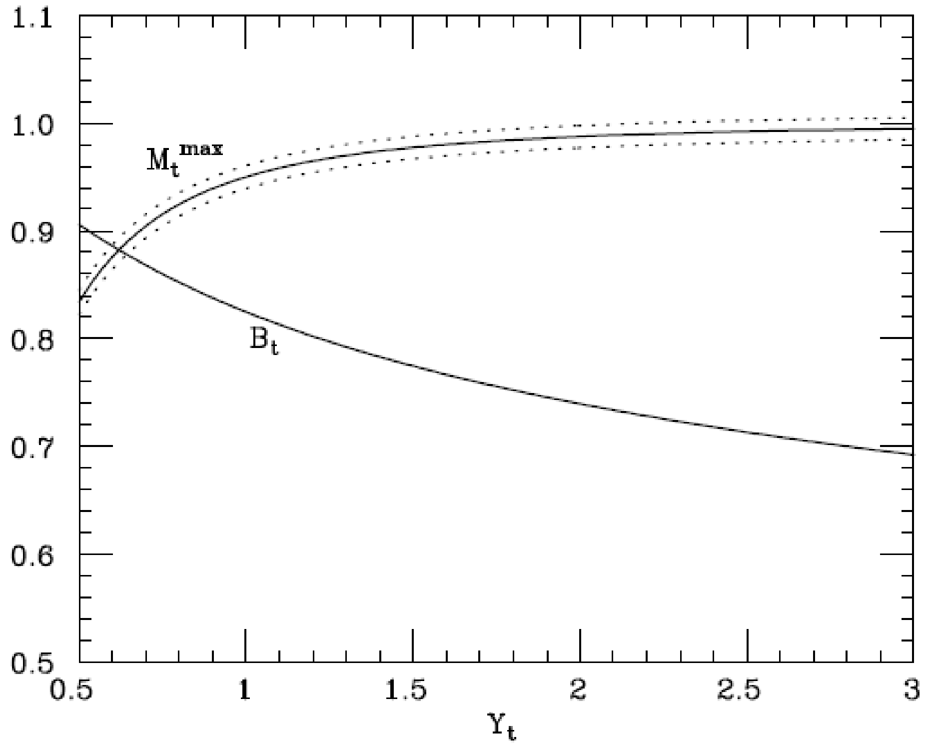



Figure 4: We see that for $Y_t$ varyng from a lower limit to its upper limit $B_t$ decreases from 0.9 to 0.7. This is very important physical fact that we will use upon our calculations. Now let us derive and list fermion mass fractions with RG coefficients and put it on the table [1].

see that for $Y_t$ varyng from a lower limit to its upper limit $B_t$ decreases from 0.9 to 0.7. This is very important physical fact that we will use upon our calculations. Now let us calculate and list fermion mass fractions with RG coefficients and put it on the table. However, before we do it let us see what are masses of fermions:

$$m_u(\mu) = 2.16^{+0.49}_{-0.26}\ MeV, \quad m_d(\mu) = 4.67^{+0.48}_{-0.17}\ MeV m_s(\mu) = 93^{+11}_{-5}\ MeV; \tag{74}$$

We must say that these masses are at a renormalization scale of $\mu = 2GeV$.

$$m_c(\mu) = 1.275^{+0.025}_{-0.035}\ GeV, \quad m_b(\mu) = 4.18^{+0.04}_{-0.03}\ GeV, m_t(\mu) = 163.3 \pm 0.4\ GeV; \tag{75}$$

Above the c,b and t-quark masses corresponds to the masses measured on $\mu = m_{c,b,t}$ respectively.

$$m_e = 0.51\ MeV, \quad m_\mu = 105.65\ MeV,\ m_\tau = 1776.86^{\pm 0.12}\ MeV; \tag{76}$$

All masses are taken from the particle data group [9]. We know that masses are running however, we cannot make division of them if they are measured on a different scale. What should be done is to convert all masses into one scale and then divide.

| **Renormalization Group Parameters** | | |
|---|---|---|
| $R_u = 3.33 \pm 0.07$ | $R_d = 3.25 \pm 0.07$ | $R_e = 1.49$ |
| $\eta_b = 1.52 \pm 0.04$ | $\eta_c = 2.02^{+0.16}_{-0.13}$ | $\eta_{uds} = 2.33^{+0.29}_{-0.21}$ |
| from $Y_t = 0.5$ to $Y_t = 3$, the factor $B_t$ decreases from 0.9 to 0.7 | | |

These are the parameters that ensure that we take all the masses on a scale of $M_z$ [4],[1].

Finally, let us calculate division of Yukawa constants. We use dependence in equation (71) and given masses in 74,75,76.

$$\begin{gathered}
\frac{Y_b}{Y_s} = \frac{\frac{m_b}{R_d \eta_b B_t}}{\frac{m_s}{R_d \eta_s}} = \left(50\frac{1}{B_t} \sim 53\frac{1}{B_t}\right) \\
\frac{Y_b}{Y_\tau} = \frac{\frac{m_b}{R_d \eta_b B_t}}{\frac{m_\tau}{R_e}} = \frac{4.18}{1.776}\frac{R_e}{R_d \eta_b B_t} \sim 0.77 B_t^{-1} \\
\frac{Y_c}{Y_t} = \frac{\frac{m_c}{R_u \eta_c B_t^3}}{\frac{m_t}{R_u B_t^6}} = 0.0078\frac{B_t^3}{\eta_c} \sim 0.0039 B_t^3 \\
\frac{Y_\mu}{Y_\tau} = \frac{m_\mu}{m_\tau} = \frac{1}{16.824} \sim 0.059
\end{gathered} \tag{77}$$

As you can see some fractions of Yukawa couplings depend on our $B_t$. In the future we will try to create model that will take into account this fact.

Furthermore, let us make a table of mass divisions below, that might be convenient for the future. One must say that these divisions are independent of the running.

$$\frac{m_u}{m_c} = 0.001976, \quad \frac{m_u}{m_d} = 0.48^{+0.07}_{-0.08},$$
$$\frac{m_b}{m_s} = 53.07, \quad \frac{m_b}{m_c} = 4.528 \pm 0.054, \quad \frac{m_s}{m_u} = 43.2,$$
$$\frac{m_s}{m_d} = 20.2, \quad \frac{m_b}{m_d} = 1072, \quad \frac{m_c}{m_t} = 0.003945,$$
$$\frac{m_\mu}{m_e} = 206.76, \quad \frac{m_\tau}{m_\mu} = 16.824, \quad \frac{m_\tau}{m_e} = 3484.03. \tag{78}$$

## 6.2 Testing Model (Exchange with 10)

At first we introduce only processes where we have exchange of 10. From the propagators (eq: 69) $3 \times 3$ matrices have the form:

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B_u \end{pmatrix} + \begin{pmatrix} 0 & \epsilon_u c \Sigma_u^A A_u & \epsilon_u s \Sigma_u^A A_u \\ -\epsilon_u c \Sigma_u^A A_u & B_u \epsilon_u \Sigma_u^s s^2 & -B_u \epsilon_u \Sigma_u^s sc \\ -\epsilon_u s \Sigma_u^A A_u & -B_u \epsilon_u \Sigma_u^s sc & 2B_u \epsilon_u \Sigma_u^s c^2 \end{pmatrix}$$
$$Y_d = \begin{pmatrix} 0 & A_d & 0 \\ -A_d & 0 & 0 \\ 0 & 0 & B_d \end{pmatrix} + \begin{pmatrix} 0 & A_d \epsilon_d \Sigma_d^A c & \epsilon_d \Sigma_d^A A_d s \\ -A_d \epsilon_d \Sigma_d^A c & B_d \epsilon_d \Sigma_d^s s^2 & -B_d \epsilon_d \Sigma_d^s sc \\ -A_d \epsilon_d \Sigma_d^A s & -B_d \epsilon_d \Sigma_d^s sc & B_d \epsilon_d \Sigma_d^s c^2 \end{pmatrix} \tag{79}$$
$$Y_l = \begin{pmatrix} 0 & A_l & 0 \\ -A_l & 0 & 0 \\ 0 & 0 & B_l \end{pmatrix} + \begin{pmatrix} 0 & A_l \epsilon_l \Sigma_l^A c & A_l \epsilon_l \Sigma_l^A s \\ -A_l \epsilon_l \Sigma_l^A c & B_l \epsilon_l \Sigma_l^s s^2 & -B_l \epsilon_l \Sigma_l^s sc \\ -A_l \epsilon_l \Sigma_l^A s & -B_l \epsilon_l \Sigma_l^s sc & B_l \epsilon_l \Sigma_l^s c^2 \end{pmatrix}$$

Where $\Sigma$ and $\Sigma^A$ are Clebsch-Gordan coefficients for symmetric and anti-symmetric part of the matrix, $\epsilon$ is the approximation parameter and $s$-$c$ sine and cosine for rotational angle of sextet and triplet. $\Sigma_u^s = \frac{3a+b}{4}$, $\Sigma_d^s = \frac{a+b}{2}$, $\Sigma_l^s = \frac{2b}{2}$,$\Sigma_u^A = \frac{a-b}{2}$,$\Sigma_d^A = \frac{a+b}{2}$,$\Sigma_l^A = \frac{2b}{2}$ (a and b from eq: 70). As you can see, down-quark and charged-lepton matrices does not have the symmetric "Fritzsch texture" in the first approx. Otherwise, we would have $tan\theta_{23}^d = \left(\frac{m_s}{m_b}\right)$ and $tan\theta_{23}^l = \left(\frac{m_\mu}{m_\tau}\right)$ which both do not satisfy CKM-first is too big second too small [1]. In our problem $A \ll B$, consequently, for the first approximation in Yukawa matrices (eq: 79) we can neglect terms with parameters A and rewrite $3 \times 3$ matrix into the form of $2 \times 2$

$$Y_u = B_u \begin{pmatrix} \epsilon \Sigma_u s^2 & -\epsilon \Sigma_u sc \\ -\epsilon \Sigma_u sc & 1 + \epsilon \Sigma_u c^2 \end{pmatrix}$$
$$Y_d = B_d \begin{pmatrix} \epsilon \Sigma_d s^2 & -\epsilon \Sigma_d sc \\ -\epsilon \Sigma_d sc & 1 + \epsilon \Sigma_d c^2 \end{pmatrix} \tag{80}$$
$$Y_l = B_l \begin{pmatrix} \epsilon \Sigma_l s^2 & -\epsilon \epsilon_l sc \\ -\epsilon \Sigma_e sc & 1 + \epsilon \Sigma_e c^2 \end{pmatrix}$$

Now let us calculate determinants and traces of square matrices.

$$\begin{aligned} B_u^2 s^2 \epsilon_u \Sigma_u &= Y_t Y_c', \quad B_u^2(1+2\epsilon_u\Sigma_u c^2) \approx Y_t^2. \\ B_d^2 s^2 \epsilon_d \Sigma_d &= Y_b Y_s', \quad B_d^2(1+2\epsilon_d\Sigma_d c^2) \approx Y_b^2. \\ B_l^2 s^2 \epsilon_l \Sigma_l &= Y_\tau Y_\mu', \quad B_l^2(1+2\epsilon_l\Sigma_l c^2) \approx Y_\tau^2. \end{aligned} \tag{81}$$

So, determinants on the left side and traces on the right. As you can see Yukawa couplings have prime on them. **This is because $(2,2)$ terms in $3 \times 3$ matrices have small corrections and with this denotion we wanted to emphasize that**. This small correction does not have any meaning in the first approximation-

$$Y_2' = Y_2 - Y_1;$$

However we will use it at the end so we must remember that. Now let us make division of determinants to the traces.

$$\begin{aligned} \frac{s^2\epsilon_u\Sigma_u}{1+2\epsilon_u\Sigma c^2} &= \frac{Y_c'}{Y_t} \\ \frac{s^2\epsilon_d\Sigma_d}{1+2\epsilon_d\Sigma c^2} &= \frac{Y_s'}{Y_b} \\ \frac{s^2\epsilon_l\Sigma_l}{1+2\epsilon_l\Sigma_l c^2} &= \frac{Y_\mu'}{Y_\tau} \end{aligned} \tag{82}$$

for the first and second ones in equation: (82) we can write that

$$\begin{aligned} \frac{Y_c'}{Y_t}/\frac{Y_s'}{Y_b} &= \frac{\Sigma_u}{\Sigma_d}(1+2(\epsilon_d\Sigma_d - \epsilon_u\Sigma_u)c^2); \\ \frac{Y_\mu'}{Y_\tau}/\frac{Y_s'}{Y_b} &= \frac{\Sigma_l}{\Sigma_d}(1+2(\epsilon_d\Sigma_d - \epsilon_l\Sigma_l)c^2); \end{aligned} \tag{83}$$

It is obvious that down quarks and leptons are come from the same diagram, therefore we can say that $B_d = B_l$. Consequently, there is a possibility to write third equation from eq: (81):

$$\frac{Y_b}{Y_\tau} = 1 + (\epsilon_d\Sigma_d - \epsilon_l\Sigma_l)c^2; \tag{84}$$

Now division of the Yukawa couplings is given in equation (77), therefore we can put them into the equations (83) and (84).

$$\begin{aligned} 0.2B_t^2 &= \frac{\Sigma_u}{\Sigma_d}(1+2(\epsilon_d\Sigma_d-\epsilon_u\Sigma_u)c^2); \\ \frac{3.11}{B_t} &= \frac{\Sigma_l}{\Sigma_d}(1+2(\epsilon_d\Sigma_d-\epsilon_l\Sigma_l)c^2); \\ \frac{0.77}{B_t} - 1 &= (\epsilon_d\Sigma_d-\epsilon_l\Sigma_l)c^2; \end{aligned} \tag{85}$$

Now, form the last two equations in (85) we can get:

$$\begin{aligned} \frac{3.11}{B_t} &= \frac{\Sigma_l}{\Sigma_d}(\frac{1.54}{B_t}-1) \Rightarrow \\ B_t &= 1-3.11\frac{\Sigma_d}{\Sigma_l} \end{aligned} \tag{86}$$

Now let us put values of $\Sigma_{u,d,l}$ in the previous equations and see what we get.

$$\begin{aligned} B_t = 1-3.11\frac{\Sigma_d}{\Sigma_l} &= 1-3.11\frac{a+b}{2b}; \\ \frac{0.77}{B_t}-1 &= \epsilon\frac{a-b}{2}c^2; \\ 0.2B_t^2 = \frac{3a+b}{2(a+b)}\left(1-\epsilon\left(\frac{3a+b}{4}-\frac{a+b}{2}\right)c^2\right)&; \end{aligned} \tag{87}$$

where $\epsilon_u = \epsilon_d = \epsilon_l \equiv \epsilon$. Reason for that is that inner operator is the same for all these diagrams.Let us solve this system of 3 equations. From the second equation in (87) we define cosine and put in the third one that gets a form:

$$0.4B_t^2 = \frac{3a+b}{(a+b)}\left(1-\epsilon\frac{a-b}{2}\frac{2(0.77-B_t)}{\epsilon(a-b)B_t}\right) \tag{88}$$

Now let us define b=1. We want that matrix: (69) to be traceless, therefore, for it, $a$ must be $a=-\frac{2}{3}$ but we will not use this condition because it is the theory that fixes $\Sigma$ and theory can be modified. Putting just value $b$ in the equation (87) and then using our formula to put in the last one will give us the form that will depend only on $B_t$. At the end of the day, we can put every logical values in the last equation (88).

$$\begin{aligned} 0.4B_t^2 &= \frac{3a+1}{(a+1)}\left(1-\frac{(0.77-B_t)}{B_t}\right) \\ a &= \frac{1-B_t}{1.555}-1; \Rightarrow \\ 0.4B_t^3(B_t-1) &= (3B_t+0.11)(2B_t-0.77) \end{aligned} \tag{89}$$

doing the last step gives us polynomial equation for $B_t$

$$0.4B_t^4 - 0.4B_t^3 - 6B_t^2 + 2.09B_t + 0.0847 = 0 \tag{90}$$

We have four real solutions:

$$B_t \sim -0.04 \quad B_t \sim 0.38 \quad B_t \sim -3.58 \quad B_t \sim 4.24 \tag{91}$$

So, that means that in physically correct area this model does **not** work. Consequently, we cannot use just 10 exchange and describe model explicitly. However, just adding exchange of 5-s will not be good for a model because it will introduce new coupling constant and new unknown parameters, therefore we should search for mode advanced model and unification. This is $SO(10)$, it coalesces in itself, everything that we want and coupling constant is same for everything. To discriminate between ups, downs and leptons we will introduce new sigma $\Sigma$ that will be 45, and this sigma will give us Clebschs that will give us hierarchy between fermions.

## 6.3 Introducing Higher Symmetry SO(10)

The main Advantage of $SO(10)$ with respect to SU(5) is that all the known SM fermions plus three right handed neutrinos fit into three copies of the 16 dimensional spinorial representation of $SO(10)$. Important branching rule for this 16 is:

$$16 \to 10 \oplus \bar{5} \oplus 1 \tag{92}$$

The $SO(10)$ symmetry is spontaneously broken to $SU(5)$ by the Higgs supermultiplet and then we have breaking of $SU(5)$ into Standard Model. As it was in MSSM here we also have two Higgses that make possible to break the symmetry and they are contained in a superfield 10. Now let us describe in more details ingredients of the model. For simplicity we assume that all vertical Higgses are singlets of $SU(3)$ and all horizontal Higgses are singlets of $SO(10)$. Vertical is just a name for Higgses of $SO(10)$. In particular the vertical sector of $SO(10)$ includes chiral superfields in the representations 54 45, 16 and $\bar{16}$, needed for the breaking down this to $SU(3) \times SU(2) \times U(1)$ and 10-plets for the electroweak symmetry breaking and the mass generation. Now for the horizontal parts we have triplet, sextet and octet representations. We, therefore, introduce anti-sextet and triplet combinations as it was in the previous case and also some adjoint representation. One should emphasize that besides the matter superfields $(16, 3)$ we introduce a number of vector-like representations as we did for $SU(5)$ these are needed for the fermion generation. Algorithm is nearly the same, these states mediate the see-saw like diagrams and hence the quark and lepton Yukawa structures emerge after integrating them out.

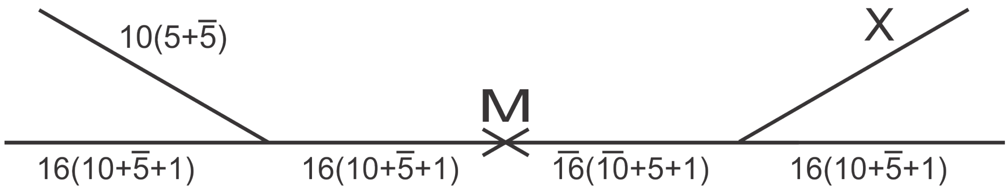
Let us talk one of the main things in our talk and emphasize some details. $SU(3)_H$ Higgs part is a little bit sophisticated. We are now taking disoriented "vectors". However, we fix the norm of them. By introducing specific type of superfields counting F terms and fixing parameters will show us how we fix a norm. Consequently we introduce $\mathcal{W}$, that is:

$$\mathcal{W}(\chi) = I(\chi\bar{\chi} - \lambda^2) \tag{93}$$

This tells us that I (that is some singlet), must be 0. and it fixes the norm of $\chi$. We do the same for the triplet. At the end, int this model we also are introducing two sextets and 1 triplet, however, initial triplet does not have just a third direction it has some component on the second direction too, therefore we have some change. To be precise, let us show the matrices:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & B \end{pmatrix} \begin{pmatrix} 0 \\ AS_\bullet \\ AC_\bullet \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & s^2B & -scB \\ 0 & -scB & c^2B \end{pmatrix} \begin{pmatrix} 0 \\ As \\ Ac \end{pmatrix} \tag{94}$$

so you can see two sextets and 1 triplet! Now we have to depict correct process for our model and that is all. If you remember we used diagrams for $SU(5)$ for exchange of 10,in the first approximation it was just 2 diagrams (+2 if we consider 5 exchange too). However, introducing $SO(10)$ merged everything and now we will have one simple process, exchanging by 16: It is




evident now that coupling constant is identical for each process that we had in $SU(5)$ because they are united in 16. For operators now we can draw the similar diagram as we did at the beginning of this section:

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ |
|---|---|---|---|
| $q$ | 0 | $\chi^T$ | $H$ |
| $Q$ | $H$ | $M_{10}$ | 0 |
| $U$ | $\chi$ | 0 | $M_{10}$ |

Yukawa matrix after diagonalization takes the form:

$$Y_u = \chi^T M_{10}^{-1} + M_{10}^{-1}\chi^T = \left(\chi + \chi^T\right) M_{10}^{-1} \tag{95}$$

Now let us see for down quarks:

| $\therefore$ | $d^c$ | $Q^c$ | $D^c$ |
|---|---|---|---|
| $q$ | $0$ | $\chi^T$ | $\bar{H}$ |
| $Q$ | $\bar{H}$ | $M_{10}$ | $0$ |
| $D$ | $\chi$ | $0$ | $M_5$ |

In this matrix you can see that there are two types of masses, one that comes from 10 exchange and one from exchange of 5. So, Yukawa matrix after diagonalization becomes:

$$Y_d = \chi^T M_{10}^{-1} + M_5^{-1}\chi \tag{96}$$

For leptons everything is settled because we know that $Y_l = Y_d^T$. Moreover, as we already mentioned [18] $16 = 1(-5) + \bar{5}(3) + 10(-1)$ where numbers in parenthesis are hypercharges that are connected to the masses heavy particles. Because of these charges we consider a case where $M_5 = -3M_{10}$. So, If we consider all of these statements we can write for a down quarks ( and leptons):

$$Y_d = \left(\chi^T - \frac{1}{3}\chi\right) M_{10}^{-1} \tag{97}$$

What we have calculated in this section so far was the a simple process, we now want to add another exchange set which will introduce new sets of exchange particles with mass $M'_{10}$ and $M'_5$. Furthermore, we have two new fields one is the $\chi'$ that has the same direction as the old but rotated *chi* (rotation happens with an angle $\alpha$) and we have field $\Sigma$. This field interacts with $16 \times \bar{16}$ that can be decomposed as $1 + 45 + 210$ [18]. For our problem 1 does not change anything, so we use 45. Ultimately, $\Sigma$ can be a combination of 1 and 24, because $45 = 1(0) + 24(0) + 10(4) + \bar{10}(-4)$. We write it as $\Sigma = \Sigma_x + \Sigma_y = x\mathbf{X} + Y_{hcharge}$. Here $X$ is unitary matrix.

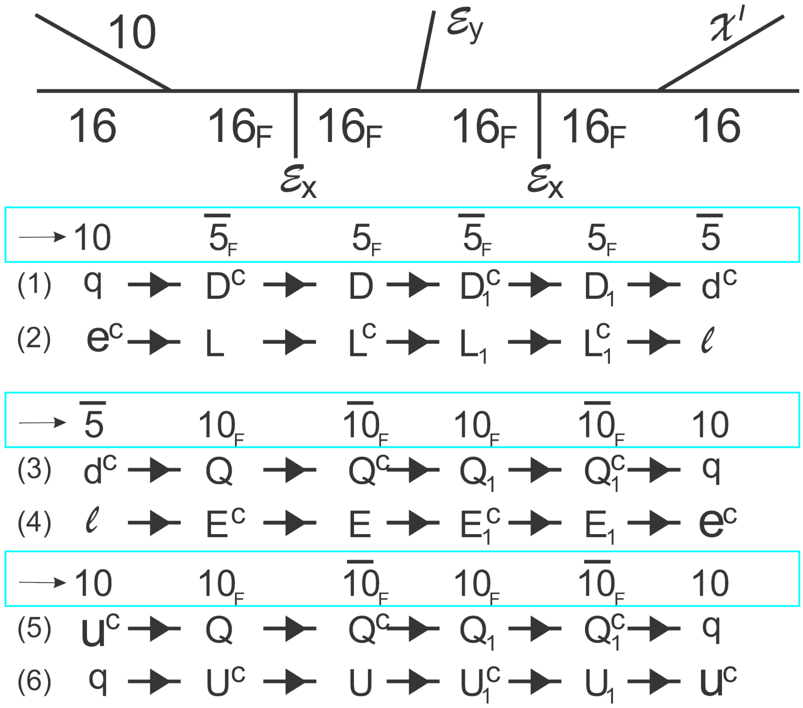



Figure 5: All process with exchange of $\bar{5}_F$ and $10_F$

So,$\Sigma_y$ is between 16 and $\bar{16}$. Each transition has been drawn, now we find which particle goes into which and from it get hypercharges. So decmposition of $\bar{5}$ and 10 are [18]:

$$\bar{5} = \left(\bar{3}, 1, +\frac{1}{3}\right) \oplus \left(1, 2, -\frac{1}{2}\right)$$
$$10 = (5 \otimes 5)_A = \left(\bar{3}, 1, -\frac{2}{3}\right) \oplus \left(\bar{3}, 2, +\frac{1}{6}\right) \oplus \left(\bar{1}, 1, +1\right) \tag{98}$$

Consequently, from it we know hypercharges of particles:

$$q \sim (3, 2, +\frac{1}{6}), \quad \ell \sim (1, 2, -\frac{1}{2}),$$
$$u^c \sim (\bar{3}, 1, -\frac{2}{3}), \quad d^c \sim (\bar{3}, 1, +\frac{1}{3}), \quad e^c \sim (1, 1, +1)). \tag{99}$$

To write everything explicitly we have to create two 5×5 matrices, one for up quarks and another for down for leptons we can make some implications easily.

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ | $U_1^c$ | $Q_1^c$ |
|---|---|---|---|---|---|
| $q$ | 0 | $\chi_u^T$ | $H$ | 0 | $\chi_u'^T$ |
| $Q$ | $H$ | $M_{10}$ | 0 | 0 | 0 |
| $U$ | $\chi_u$ | 0 | $M_{10}$ | $\Sigma_{U-U_1^c}$ | 0 |
| $U_1$ | $\chi_u'$ | 0 | 0 | $M_5'$ | 0 |
| $Q_1$ | 0 | $\Sigma_{Q^c-Q_1}$ | 0 | 0 | $M_{10}'$ |

After integrating:

| $\therefore$ | $u^c$ | $Q^c$ | $U^c$ | $U_1^c$ | $Q_1^c$ |
|---|---|---|---|---|---|
| $q$ | 0 | $\chi_u^T + \triangle\chi_u^T$ | $H$ | 0 | 0 |
| $Q$ | $H$ | $M_{10}$ | 0 | 0 | 0 |
| $U$ | $\chi + \triangle\chi_u$ | 0 | $M_{10}$ | 0 | 0 |
| $U_1$ | 0 | 0 | 0 | $M_{10}'$ | 0 |
| $Q_1$ | 0 | 0 | 0 | 0 | $M_{10}'$ |

where $\triangle\chi_u = \Sigma_{U-U_1^c} M_{10}'^{-1}\chi_u'$ and $\triangle\chi_u^T = \chi_u'^T M_{10}^{-1}\Sigma_{Q^c-Q_1}$. If we put values of $Sigma$ we will get:

$$\chi \to \chi + \triangle\chi = \chi + (x + \frac{2}{3})M_{10}'^{-1}\chi'$$
$$\chi^T \to \chi^T + \triangle\chi^T = \chi^T + (x - \frac{1}{6})M_{10}'^{-1}\chi'^T \tag{100}$$

Consequently, Yukawa matrix for up quarks will have a form:

$$Y_u = \left(\chi^T + \chi + x(\chi' + \chi'^T)M_{10}'^{-1} + \left(\frac{2}{3}\chi' - \frac{1}{6}\chi'^T\right) M_{10}'^{-1}\right) M_{10}^{-1} \tag{101}$$

Okay, it is time for down part, however here we have also exchange of heavy particles with mass $M_5'$ which might have different charge $X$ with respect to other heavy particles with mass $M_1'0$. I want to convey that if we have same exchange with 16 then for sure we have $M_5' = -3M_1'0$ but also it is possible to have heavier particle exchange where $M_5' = M_1'0$. So two cases that must be considered.

| $\therefore$ | $d^c$ | $Q^c$ | $D^c$ | $D_1^c$ | $Q_1^c$ |
|---|---|---|---|---|---|
| $q$ | 0 | $\chi_d^T$ | $\bar{H}$ | 0 | $\chi_d'^T$ |
| $Q$ | $\bar{H}$ | $M_{10}$ | 0 | 0 | 0 |
| $D$ | $\chi_d$ | 0 | $M_5$ | $\Sigma_{D-D_1^c}$ | 0 |
| $D_1$ | $\chi_d'$ | 0 | 0 | $M_5'$ | 0 |
| $Q_1$ | 0 | $\Sigma_{Q^c-Q_1}$ | 0 | 0 | $M_{10}'$ |

After integrating:

| $\therefore$ | $d^c$ | $Q^c$ | $D^c$ | $D_1^c$ | $Q_1^c$ |
|---|---|---|---|---|---|
| $q$ | $0$ | $\chi_d^T + \triangle\chi_d^T$ | $\bar{H}$ | $0$ | $\chi_d'^T$ |
| $Q$ | $\bar{H}$ | $M_{10}$ | $0$ | $0$ | $0$ |
| $D$ | $\chi + \triangle\chi_d$ | $0$ | $M_5$ | $0$ | $0$ |
| $D_1$ | $0$ | $0$ | $0$ | $M_5'$ | $0$ |
| $Q_1$ | $0$ | $0$ | $0$ | $0$ | $M_{10}'$ |

where $\triangle\chi_d = \Sigma_{D-D_1^c} M_5'^{-1}\chi_d'$ and $\triangle\chi_d^T = \chi_d'^T M_{10}^{-1}\Sigma_{Q^c-Q_1}$. Putting values of $Sigma$:

$$\begin{aligned} \chi \to \chi + \triangle\chi = \chi + ((-3)x - \frac{1}{3})M_5'^{-1}\chi' \\ \chi^T \to \chi^T + \triangle\chi^T = \chi^T + (x - \frac{1}{6})M_{10}'^{-1}\chi'^T \end{aligned} \tag{102}$$

Consequently, Yukawa matrix for down quarks will have a form:

$$Y_d = \left(\chi^T - \frac{1}{3}\chi + x\left(M_{10}'^{-1}\chi'^T + M_5'^{-1}\chi'\right) + \frac{1}{9}M_5'^{-1}\chi' - \frac{1}{6}M_{10}'^{-1}\chi'^T\right) M_{10}^{-1} \tag{103}$$

For the first case, when $M_5' = -3M_{10}'$

$$Y_d = \left(\chi^T - \frac{1}{3}\chi + x\left(\chi'^T - \frac{1}{3}\chi'\right) M_{10}'^{-1} - \left(\frac{1}{27}\chi' + \frac{1}{6}\chi'^T\right) M_{10}'^{-1}\right) M_{10}^{-1} \tag{104}$$

For the second case, when $M_5' = M_{10}'$

$$Y_d = \left(\chi^T - \frac{1}{3}\chi + x\left(\chi'^T + \chi'\right) M_{10}'^{-1} + \left(\frac{1}{9}\chi' - \frac{1}{6}\chi'^T\right) M_{10}'^{-1}\right) M_{10}^{-1} \tag{105}$$

Form of a $\chi$ and $\chi'$ is the same as it was in previous model. They are comprised of sextet and triplet. Let us make such kind of denotion: $\frac{\chi_{33}}{M_{10}} \equiv B$, $\frac{\chi_{12}}{M_{10}} \equiv A$ and $\frac{\langle\Sigma\rangle}{M_{10}'} \equiv \epsilon$. Now let us put explicit form to get mass matrices:

$$\begin{aligned} Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2B \end{pmatrix} + \\ x\epsilon \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2Bs^2 & -2Bsc \\ 0 & -2Bsc & 2Bc^2 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & \frac{5}{6}Ac & \frac{5}{6}As \\ -\frac{5}{6}Ac & \frac{1}{2}Bs^2 & -\frac{1}{2}Bsc \\ -\frac{5}{6}As & -\frac{1}{2}Bsc & \frac{1}{2}Bc^2 \end{pmatrix} \Rightarrow \end{aligned} \tag{106}$$

$$Y_u = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2B \end{pmatrix} + \epsilon \begin{pmatrix} 0 & \frac{5}{6}Ac & \frac{5}{6}As \\ -\frac{5}{6}Ac & (2x + \frac{1}{2})Bs^2 & -(2x + \frac{1}{2})Bsc \\ -\frac{5}{6}As & -(2x + \frac{1}{2})Bsc & (2x + \frac{1}{2})Bc^2 \end{pmatrix}$$

We did not expect much of a change in Up part, inasmuch as there are no 5 exchange in this part. Okay, let us move on an important part-down quark mass matrix (For the case $M'_5 = -3M'_{10}$).

$$Y_d = \begin{pmatrix} 0 & -\frac{4}{3}AC_\bullet & -\frac{4}{3}AS_\bullet \\ \frac{4}{3}AC_\bullet & 0 & 0 \\ \frac{4}{3}AS_\bullet & 0 & \frac{2}{3}B \end{pmatrix} +$$

$$x\epsilon \begin{pmatrix} 0 & -\frac{4}{3}Ac & -\frac{4}{3}As \\ \frac{4}{3}Ac & \frac{2}{3}Bs^2 & -\frac{2}{3}Bsc \\ \frac{4}{3}Ac & -\frac{2}{3}Bsc & \frac{2}{3}Bc^2 \end{pmatrix} - \epsilon \begin{pmatrix} 0 & -\frac{7}{54}Ac & -\frac{7}{54}As \\ \frac{7}{54}Ac & \frac{11}{54}Bs^2 & -\frac{11}{54}Bsc \\ \frac{7}{54}As & -\frac{11}{54}Bsc & \frac{11}{54}Bc^2 \end{pmatrix} \Rightarrow \tag{107}$$

$$Y_d = \begin{pmatrix} 0 & -\frac{4}{3}AC_\bullet & -\frac{4}{3}AS_\bullet \\ \frac{4}{3}AC_\bullet & 0 & 0 \\ \frac{4}{3}AS_\bullet & 0 & \frac{2}{3}B \end{pmatrix} +$$

$$\epsilon \begin{pmatrix} 0 & -(\frac{4}{3}x - \frac{7}{54})Ac & -(\frac{4}{3}x - \frac{7}{54})As \\ (\frac{4}{3}x - \frac{7}{54})Ac & (\frac{2}{3}x - \frac{11}{54})Bs^2 & -(\frac{2}{3}x - \frac{11}{54})Bsc \\ (\frac{4}{3}x - \frac{7}{54})As & -(\frac{2}{3}x - \frac{11}{54})Bsc & (\frac{2}{3}x - \frac{11}{54})Bc^2 \end{pmatrix}$$

For the case $M'_5 = M'_{10}$.

$$Y_d = \begin{pmatrix} 0 & -\frac{4}{3}AC_\bullet & -\frac{4}{3}AS_\bullet \\ \frac{4}{3}AC_\bullet & 0 & 0 \\ \frac{4}{3}AS_\bullet & 0 & \frac{2}{3}B \end{pmatrix} +$$

$$\epsilon \begin{pmatrix} 0 & \frac{5}{18}Ac & \frac{5}{18}As \\ -\frac{5}{18}Ac & (2x - \frac{1}{18})Bs^2 & -(2x - \frac{1}{18})Bsc \\ -\frac{5}{18}As & -(2x - \frac{1}{18})Bsc & (2x - \frac{1}{18})Bc^2 \end{pmatrix}$$

Now, what about lepton part. We knew that without inserting $\Sigma$ lepton part was just a transpose, however, after this $\Sigma$ enters in a game we have different hypercharges and we cannot make a simple transpose for the second part of the matrix. We must go back to the $5 \times 5$ matrices and to the transitions and see what is changing. So,it is evident that the $M_5$ and $M'_5$ switch places with $M_{10}$ and $M'_{10}$. Moreover, $\triangle\chi$ is changed also in a sense that $\Sigma$ is different.

$$\triangle\chi_l = \Sigma_{E-E_1^c} M'^{-1}_{10}\chi' = (x-1)M'^{-1}_{10}\chi';$$
$$\triangle\chi_l^T = \chi'^T M'^{-1}_5 \Sigma_{L^c-L_1} = \left((-3)x + \frac{1}{2}\right)\chi'^T M'^{-1}_5; \tag{108}$$

So for leptons:

$$Y_l = \left(\chi - \frac{1}{3}\chi^T + x\left(\chi' M'^{-1}_{10} + \chi'^T M'^{-1}_5\right) - \left(\frac{1}{6}\chi'^T M'^{-1}_5 + \chi' M'^{-1}_{10}\right)\right) M^{-1}_{10} \tag{109}$$

For the case $M_5' = -3M_{10}'$

$$Y_l = \begin{pmatrix} 0 & \frac{4}{3}AC_\bullet & \frac{4}{3}AS_\bullet \\ -\frac{4}{3}AC_\bullet & 0 & 0 \\ -\frac{4}{3}AS_\bullet & 0 & \frac{2}{3}B \end{pmatrix} + \epsilon \begin{pmatrix} 0 & (\frac{4}{3}x - \frac{19}{18})Ac & (\frac{4}{3}x - \frac{19}{18})As \\ -(\frac{4}{3}x - \frac{19}{18})Ac & (\frac{2}{3}x - \frac{17}{18})Bs^2 & -(\frac{2}{3}x - \frac{17}{18})Bsc \\ -(\frac{4}{3}x - \frac{19}{18})As & -(\frac{2}{3}x - \frac{17}{18})Bsc & (\frac{2}{3}x - \frac{17}{18})Bc^2 \end{pmatrix} \quad (110)$$

For the case $M_5' = M_{10}'$

$$Y_l = \begin{pmatrix} 0 & \frac{4}{3}AC_\bullet & \frac{4}{3}AS_\bullet \\ -\frac{4}{3}AC_\bullet & 0 & 0 \\ -\frac{4}{3}AS_\bullet & 0 & \frac{2}{3}B \end{pmatrix} + \epsilon \begin{pmatrix} 0 & \frac{5}{6}Ac & \frac{5}{6}As \\ -\frac{5}{6}Ac & (2x - \frac{7}{6})Bs^2 & -(2x - \frac{7}{6})Bsc \\ -\frac{5}{6}As & -(2x - \frac{7}{6})Bsc & (2x - \frac{7}{6})Bc^2 \end{pmatrix} \quad (111)$$

For convenience, let us split our problem in two cases. Further calculations will be held separately.

### 6.3.1 When $M_5' = M_{10}'$

Consequently,

$$Y_u = \begin{pmatrix} 0 & \frac{5}{6}A\epsilon c & \frac{5}{6}A\epsilon s \\ -\frac{5}{6}A\epsilon c & (2x + \frac{1}{2})B\epsilon s^2 & -(2x + \frac{1}{2})B\epsilon sc \\ -\frac{5}{6}A\epsilon s & -(2x + \frac{1}{2})B\epsilon sc & (2 + (2x + \frac{1}{2})\epsilon c^2)B \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & -(\frac{4}{3}C_\bullet - \frac{5}{18}\epsilon c)A & -(\frac{4}{3}S_\bullet - \frac{5}{18}\epsilon s)A \\ (\frac{4}{3}C_\bullet - \frac{5}{18}\epsilon c)A & (2x - \frac{1}{18})B\epsilon s^2 & -(2x - \frac{1}{18})B\epsilon sc \\ (\frac{4}{3}S_\bullet - \frac{5}{18}\epsilon s)A & -(2x - \frac{1}{18})B\epsilon sc & (\frac{2}{3} + (2x - \frac{1}{18})\epsilon c^2)B \end{pmatrix} \quad (112)$$

$$Y_l = \begin{pmatrix} 0 & (\frac{4}{3}C_\bullet + \frac{5}{6}\epsilon c)A & (\frac{4}{3}S_\bullet + \frac{5}{6}\epsilon s)A \\ -(\frac{4}{3}C_\bullet + \frac{5}{6}\epsilon c)A & (2x - \frac{7}{6})B\epsilon s^2 & -(2x - \frac{7}{6})B\epsilon sc \\ -(\frac{4}{3}S_\bullet + \frac{5}{6}\epsilon s)A & -(2x - \frac{7}{6})B\epsilon sc & (\frac{2}{3} + (2x - \frac{7}{6})\epsilon c^2)B \end{pmatrix}$$

Now let us estimate everything with the approximation of $\epsilon$. We will neglect terms with $\epsilon^2$. Let us look on the up-quark matrix. If we make (23) rotation to diagonalize it (detailed calculation in the Appendix) for (22) term we are getting $(2x + \frac{1}{2})B\epsilon s^2 + \frac{(2x+\frac{1}{2})^2 B^2 \epsilon^2 s^2 c^2}{2B + (2x+\frac{1}{2})B\epsilon c^2}$, so as you can see we have $\epsilon^2$ on the second term and we neglect it. We can do it for these three matrices and we can get Yukawa constants with $\epsilon$ approximation for charm,top,strange,bottom,mu and tau particles.

$$Y'_c \sim \left(2x + \frac{1}{2}\right) B\epsilon s^2, \quad Y_t \sim \left(2 + \left(2x + \frac{1}{2}\right) \epsilon c^2\right) B$$
$$Y'_s \sim \left(2x - \frac{1}{18}\right) B\epsilon s^2, \quad Y_b \sim \left(\frac{2}{3} + \left(2x - \frac{1}{18}\right) \epsilon c^2\right) B$$
$$Y'_\mu \sim \left(2x - \frac{7}{6}\right) B\epsilon s^2, \quad Y_\tau \sim \left(\frac{2}{3} + \left(2x - \frac{7}{6}\right) \epsilon c^2\right) B$$

In the previous section 6.1 we have calculated the Yukawa constant divisions(**in the first approximation**), so let us use them (we put values of $B_t$ from 0.7 to 0.9 respectively):

$$\frac{Y'_c}{Y_t} \sim (x + 0.25)\epsilon s^2 \sim 0.0039 B_t^3 = [0.00134 \div 0.00284]\,;$$
$$\frac{Y'_s}{Y_b} \sim 3\left(x - \frac{1}{36}\right) \epsilon s^2 \sim \frac{B_t}{50} = [0.014 \div 0.018]\,;$$
$$\frac{Y'_\mu}{Y_\tau} \sim 3\left(x - \frac{7}{12}\right) \epsilon s^2 \sim 0.059; \tag{113}$$

Now let us look on the first and second pair in equation 113.

$$\left|\frac{Y'_s}{Y_b} / \frac{Y'_c}{Y_t}\right| = 3\frac{\left(x - \frac{1}{18}\right)}{x + 0.25} = |10.45 \div 6.34| \tag{114}$$

We have to solve in case of moduli and get values for x:

$$\begin{array}{ll} (-) & x = [-0.188 \quad \div \quad -0.16] \\ (+) & x = [-0.362 \quad \div \quad -0.5] \end{array} \tag{115}$$

From equation (113) let us use third-second pair and see which values of $x$ satisfies it:

$$\left|\frac{Y'_\mu}{Y_\tau} / \frac{Y'_s}{Y_b}\right| = \frac{\left(x - \frac{7}{12}\right)}{x - \frac{1}{36}} = \frac{3.15}{B_t} = [4.5 \div 3.5]$$
$$x \in [-0.1309 \div -0.194] \tag{116}$$

As you can see our solution in equation (135) fully covers area of valid $x$ in equation (115). Therefore now we can find the point where these lines interact and give us the solutions: $x$ and $B_t$.

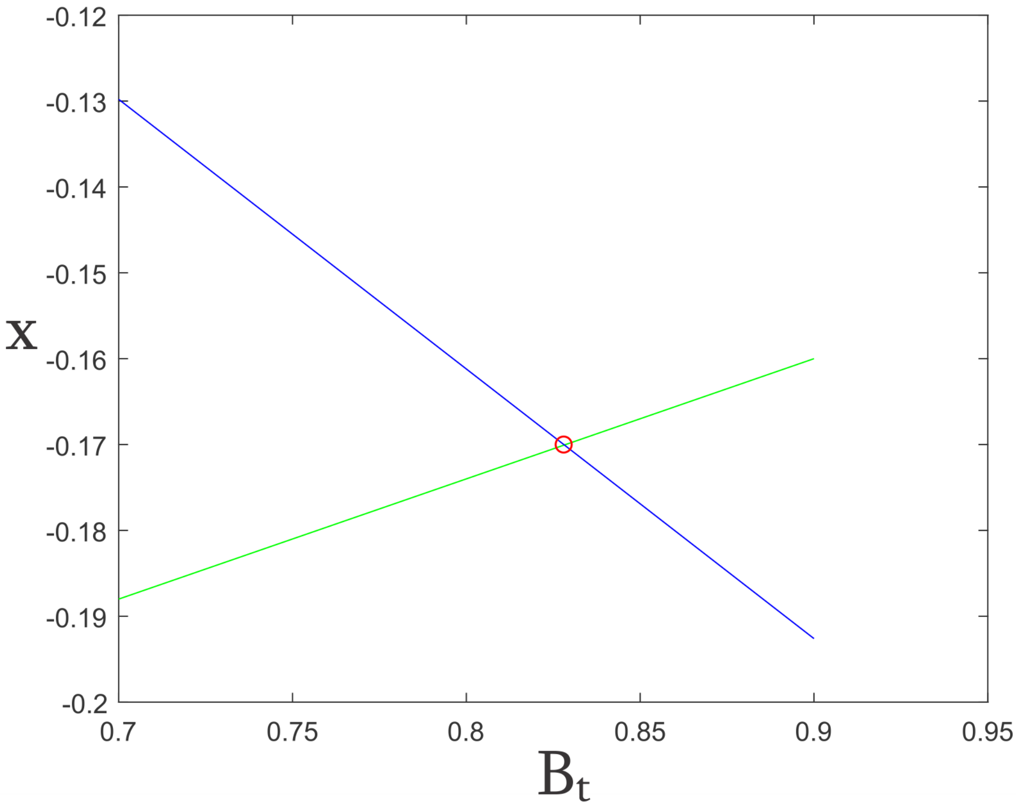



Figure 6: Dependence of $x$ on the $B_t$. Red point represents the interaction point that is $(0.826, -0.17)$.

So, so far we found $B_t$ for our model that is $B_t = 0.826$ and $x = -0.17$. Let us put these values and find exactly what are Yukawa coupling divisions:

$$\frac{Y_c'}{Y_t} \sim \frac{1}{455}, \quad \frac{Y_s'}{Y_b} \sim \frac{1}{64}, \quad \frac{Y_s'}{Y_b} / \frac{Y_c'}{Y_t} \sim 7.109$$

Now let us estimate what should be the angle $\alpha$ between the $\chi$ and $\chi'$. First of all we should say that in CKM matrix ($V_{CKM} = V_u^\dagger V_d$) contribution of up-quarks is very small so we can neglect it in the first approximation. Therefore, it can be said that $tan^d \alpha_{23} \simeq 0.042$. From the appendix (8), we can easily write the exact rotational angle for down-quarks.

$$\begin{aligned}\frac{Y_s'}{Y_b} \sim \frac{1}{64} &= 3(x-\frac{1}{36})\epsilon s^2 \\ tan(\theta_{23}) \sim 3(x &- \frac{1}{36})\epsilon sc \Rightarrow \\ tan(\alpha) = \frac{\frac{1}{50}}{0.042} &\sim 0.449\end{aligned} \tag{117}$$

So $\alpha \sim 25.74°$. Now that we know the rotational angle we can find also smallness parameter too. Just for convenience I will give a numerical results for $c^2$ and $s^2$ that are 0.8116 and 0.1883 respectively. Let us put values in the second equation of (113). So, in the first approximation we get $\epsilon \sim -0.1479$. We have found all parameters that are necessary to start process of "iteration" and make better calculations in the second approximation. It is handy to start with down quark and lepton part. Considering small term in Yukawa coupling constants division of $\tau$ and bottom quark will be:

$$\frac{Y_\tau}{Y_b} = \frac{1+3(1-\frac{7}{12})\epsilon c^2}{1+3(1-\frac{1}{18})\epsilon c^2} \sim 1.1868 \equiv Q \tag{118}$$

We have now everything to estimate mass of the bottom quark. Must not be forgotten that we have to use $R$ and $\eta$ factors to take masses from GUT to their measurement scale.

$$m_b = \frac{Y_\tau}{Q} = \frac{R_d\eta_b}{R_e}\frac{m_\tau B_t}{Q} \sim \frac{3.31}{1.1868}1.776B_t \sim 4.961B_t \sim 4.117\ GeV \tag{119}$$

So, we can say our value falls into the range of the particle data group given mass [9]! Now it is vital to "talk to" the asymmetric part of Yukawa matrix and estimate parameter A. As you can see in the down and lepton matrices we have $\epsilon$-corrections too that we have to consider. However, it must be emphasized that in the first approx. determinants of these two matrices are equal $Y_dY_sY_b = Y_eY_\mu Y_\tau$

$$\left(\frac{4}{3}-\frac{5}{18}\epsilon c\right)A \sim \sqrt{Y_dY_s}\left(\frac{4}{3}+\frac{5}{6}\epsilon c\right)A \sim \sqrt{Y_eY_\mu} \Rightarrow Y_dY_s = Y_eY_\mu\frac{1.223}{1.3684}$$

Now, introducing higher order correction into the determinant will give us:

$$Y_dY_s = Y_eY_\mu \cdot 0.947 \cdot Q \tag{120}$$

To extract $\frac{Y_d}{Y_s}$ we also use another divisions from equation (113). Let us divide second and third one:

$$\frac{Y_s - Y_d}{Y_b}/\frac{Y_\mu - Y_e}{Y_\tau} \sim \frac{B_t}{51}\cdot\frac{1}{0.059} \sim 0.276 \equiv R \tag{121}$$

We have used this equation (82).Now, let us divide eq:(120) on square of eq:(121):

$$\frac{Y_d}{Y_s} \cdot \left(1 - \frac{Y_d}{Y_s}\right)^{-2} = \frac{Y_e}{Y_\mu}\left(1 - \frac{Y_e}{Y_\mu}\right)^{-2} \frac{Q^3}{R^2} \tag{122}$$

From this equation it is easy to find our wanted division that is

$$\frac{Y_d}{Y_s} \sim \frac{1}{20.3}. \tag{123}$$

This also matches with the experimentally given number in the error range.

Let us move to the up-quark Yukawa matrix and find mass divisions there too. For ups in asymmetric part we only have $\epsilon$-correction terms, therefore for A there we have:

$$\frac{5}{6}A\epsilon c = \sqrt{Y_u Y_c} \tag{124}$$

We also will need this equation:

$$Y_c \sim B\left(2x - \frac{1}{18}\right)\epsilon c^2. \tag{125}$$

now their division equals to:

$$\frac{A}{B} \cdot \frac{5}{6}\frac{c}{s^2(2x+0.5)} = \sqrt{\frac{Y_u}{Y_c}} \tag{126}$$

If we do same for down-quark:

$$\frac{A}{B} \cdot \frac{4}{3}\frac{1}{\epsilon s^2\left(2x - \frac{1}{18}\right)} = \sqrt{\frac{Y_d}{Y_s}} \tag{127}$$

Division of eq:(126) into eq:(127) gives us:

$$\frac{Y_u}{Y_c} = \frac{25}{36} \cdot \frac{9}{16} \cdot \frac{c}{s^2(2x+0.5)} \cdot \frac{\epsilon s^2(2x - \frac{1}{18})}{4} \cdot \frac{Y_d}{Y_s} \sim 0.002 = \frac{1}{500} \tag{128}$$

This factor gets also well with experimental value. As you know we have calculated and estimated $B_t$ that is nearly 0.83. $B_t$ is a function of $Y_t$, therefore we also know it from the figure: (4). $Y_t \sim 0.91$ for calculating $tan(\beta)$ (that is $v_1/v_2$) we take division of top quark to bottom quark that is:

$$\frac{Y_b}{Y_t} = \frac{\frac{2}{3} + \left(2x - \frac{1}{18}\right)\epsilon c^2}{2 + (2x+0.5)\,\epsilon c^2} \simeq 0.3604 \equiv P \tag{129}$$

Then if we use definition of $Y_b$ and put there mass of the bottom quark that we already took we find that:

$$cos(\beta) = \frac{m_b}{PY_tR_d\eta_d \cdot 174} \Rightarrow$$
$$tan(\beta) = 68.486; \quad (130)$$

We know the $\beta$ angle too, therefore it is very easy to find $m_t$:

$$m_t = Y_tR_u(B_t)^6 \cdot 174 \cdot sin(\beta) \simeq 0.91 \cdot 3.25 \cdot (0.826)^6 \cdot 174 \cdot sin(\beta) \sim 163.421;$$

Consequently, $m_t = 163.421GeV$ is a very good theoretical value and it matches with the experiment! Now we have estimated vital parameters and we can clearly and easily get not only division of masses but also their values. Let us put all needed parameters in this equation $Y_s = B\left(2x - \frac{1}{18}\right)\epsilon s^2$. We already know $Y_t$ therefore we can easily get B coefficient from it ($B \simeq 0.4594$).

$$m_s = BR_d\eta_s \cdot 174 \cdot cos(\beta)\left(2x - \frac{1}{18}\right)\epsilon s^2 \simeq 0.0968GeV = 96.8MeV;$$
$$\frac{Y_d}{Y_s} = \frac{m_d}{m_s} \Rightarrow m_d \simeq \frac{1}{20.3} \cdot 96.8 \sim 4.768MeV; \quad (131)$$

Both are very good results if you check with experimental values [9]. Analogical strategy will be used for the upper part and we will get that:

$$m_c = BR_u\eta_c \cdot 174 \cdot sin(\beta)B_t^3\left(2x + 0.5\right)\epsilon s^2 \simeq 1.32GeV; \quad (132)$$

by the same logic we calculate up quark mass that will be $m_u \sim 2.64\ MeV$. As you can see all 6 masses that we are getting from theoretical considerations **match with experiment very well**. Now, let us calculate also $s_{12}$ and $s_13$ The main part in CKM is bottom-quark, inasmuch as up is very "stretched" and it gives little contribution, therefore one can say that $s_{12} = \sqrt{\frac{m_d}{m_s}} \sim 0.22$ that is nearly $\sim 12.8°$. For the (13) mixing we have that $|V_{ub}| = \frac{\sqrt{ds}}{b} \sim \sqrt{\frac{d}{s}} \cdot \frac{s}{b} \sim 4 \cdot 10^{-3}$. However, this good result is possible if $C_\bullet = S_\bullet = \frac{\sqrt{2}}{2}$ so it is 45°.

### 6.3.2 When $M_5' = -3M_{10}'$

So,

$$Y_u = \begin{pmatrix} 0 & \frac{5}{6}A\epsilon c & \frac{5}{6}A\epsilon s \\ -\frac{5}{6}A\epsilon c & (2x+\frac{1}{2})B\epsilon s^2 & -(2x+\frac{1}{2})B\epsilon sc \\ -\frac{5}{6}A\epsilon s & -(2x+\frac{1}{2})B\epsilon sc & (2+(2x+\frac{1}{2})\epsilon c^2)B \end{pmatrix}$$

$$Y_d = \begin{pmatrix} 0 & -(\frac{4}{3}C_\bullet + (\frac{4}{3}x - \frac{7}{54})\epsilon c)A & -(\frac{4}{3}S_\bullet + (\frac{4}{3}x - \frac{7}{54})\epsilon s)A \\ (\frac{4}{3}C_\bullet + (\frac{4}{3}x - \frac{7}{54})\epsilon c)A & (\frac{2}{3}x - \frac{11}{54})B\epsilon s^2 & -(\frac{2}{3}x - \frac{11}{54})B\epsilon sc \\ (\frac{4}{3}S_\bullet + (\frac{4}{3}x - \frac{7}{54})\epsilon s)A & -(\frac{2}{3}x - \frac{11}{54})B\epsilon sc & (\frac{2}{3} + (\frac{2}{3}x - \frac{11}{54})\epsilon c^2)B \end{pmatrix} \quad (133)$$

$$Y_l = \begin{pmatrix} 0 & (\frac{4}{3}C_\bullet + (\frac{4}{3}x - \frac{19}{18})\epsilon c)A & (\frac{4}{3}S_\bullet + (\frac{4}{3}x - \frac{19}{18})\epsilon s)A \\ -(\frac{4}{3}C_\bullet + (\frac{4}{3}x - \frac{19}{18})\epsilon c)A & (\frac{2}{3}x - \frac{17}{18})B\epsilon s^2 & -(\frac{2}{3}x - \frac{17}{18})B\epsilon sc \\ -(\frac{4}{3}S_\bullet + (\frac{4}{3}x - \frac{19}{18})\epsilon s)A & -(\frac{2}{3}x - \frac{17}{18})B\epsilon sc & (\frac{2}{3} + (\frac{2}{3}x - \frac{17}{18})\epsilon c^2)B \end{pmatrix}$$

Here we do the same procedures as we did in the last subsection-considering two equations finding all areas where our $x$ is good and then checking if for these two equations these areas cover each other.

$$\left|\frac{Y_s'}{Y_b}/\frac{Y_c'}{Y_t}\right| = \frac{\left(x - \frac{11}{36}\right)}{x + 0.25} = |10.45 \div 6.34| \quad \Rightarrow \quad (134)$$
$$(+)\ \ x \in [-0.31\ \div\ -0.35]$$
$$(-)\ \ x \in [-0.2\ \div\ -0.175]$$

$$\left|\frac{Y_\mu'}{Y_\tau}/\frac{Y_s'}{Y_b}\right| = \frac{\left(x - \frac{7}{12}\right)}{x - \frac{1}{36}} = \frac{3.15}{B_t} = [4.5 \div 3.5] \quad \Rightarrow \quad (135)$$
$$(+)\ \ x \in [-0.012\ \div\ -0.138]$$
$$(-)\ \ x \in [0.5\ \div\ 0.55]$$

As you can see neither of these areas cover each other, therefore we do not have a solution for $x$ in this model.

# 7 Conclusion

Introducing a new symmetry on a GUT-SUSY scale made possible to somehow explain hierarchy of masses between families. We created a model where for input there is used a lepton part, however you get output of quark masses and mixing angles. This is quite beautiful and thrilling.

| Output | | |
|---|---|---|
| $\frac{Y_u}{Y_c} \sim \frac{1}{500}$ | $\therefore$ | $\frac{Y_d}{Y_s} \sim \frac{1}{20.3}$ |
| $m_u(2GeV) \simeq 2.64\ MeV$ | $m_c(M_c) \simeq 1.32\ GeV$ | $m_t(M_t) \simeq 163.42GeV$ |
| $m_d(2GeV) \simeq 4.768\ MeV$ | $m_s(2GeV) \simeq 96.8\ MeV$ | $m_b(M_b) \simeq 4.117\ GeV$ |
| $s_{12} \simeq 0.22$ | $\therefore$ | $s_{13} \simeq 4 \cdot 10^{-3}$ |

# 8 Appendix

## 8.1 Seesaw Mechanism

Neutrinos are special in the Standard Model for three reasons: 1. Neutrinos have no electric charge. 2. Neutrinos are so much lighter in mass than all the other particles of the Standard Model. 3. Neutrinos are only left-handed. By linking these three apparently different facts together Murray Gell-Mann was able to propose the seesaw mechanism. The seesaw mechanism is a general phenomenon that occurs with eigenvalues of matrices of a certain form. Consider a matrix:

$$\begin{pmatrix} 0 & a \\ a & b \end{pmatrix} \tag{136}$$

which has eigenvalues

$$\lambda_+ = \frac{b}{2} + \sqrt{(\frac{b}{2})^2 - a^2} \tag{137}$$

$$\lambda_- = \frac{b}{2} - \sqrt{(\frac{b}{2})^2 - a^2} \tag{138}$$

It is needed to emphasize the fact that light eigenvalue is parametrically smaller than any of the (non-zero) elements of the matrix. The bigger that the 'larger' eigenvalue becomes, the smaller the 'lighter' eigenvalue becomes. From this property arouses the name "seesaw", one goes up, the other goes down. In reality, if there is a small eigenvalue to explain, the seesaw formula transforms it into explanation when a different eigenvalue is so big. Transforming problems this way is an effective method in science. How the seesaw mechanism is believed to work in the neutrino sector of the Standard Model is as follows.

Fermions in the Standard Model are Weyl fermions — half of a normal Dirac fermion. These Weyl fermions have a definite chirality meaning that the spin and the momentum are either perfectly aligned (so-called right-handed fermions) or perfectly anti-aligned (left-handed fermions) Weyl fermions are massless. The Standard Model fermions obviously have masses (otherwise the electron wouldn't form atoms). Fermions acquire mass in the Standard Model through electroweak symmetry breaking. When electroweak symmetry is broken by the Higgs field acquiring a vacuum expectation value, the left-handed Weyl fermions find right-handed Weyl fermions and "marry" through a mass term and become a Dirac fermion. In the Standard Model it looks like this:

$$\mathcal{L}_{mass} = (\psi \quad \psi^c) \begin{pmatrix} 0 & m \\ m & 0 \end{pmatrix} \begin{pmatrix} \psi \\ \psi^c \end{pmatrix} + h.c. \tag{139}$$

where $\psi$ is the left-handed part of the fermion species and $\psi^c$ is the charge conjugate of the right-handed part of the fermion species. The diagonal zeros of the mass matrix are forbidden by electroweak symmetry (and often by electric charge). The eigenvalues of this matrix are degenerate and equal to $m$.

Let's consider Neutrino masses. One of the ways that neutrinos are special is because there is only left-handed part of the fermion species and there is no right-handed part of the neutrino species in the Standard Model. This results in the mass term for the neutrinos in the Lagrangian to be

$$\mathcal{L}_{\nu-mass} = (\nu \quad -) \begin{pmatrix} 0 & - \\ - & - \end{pmatrix} \begin{pmatrix} \nu \\ - \end{pmatrix} + h.c. \tag{140}$$

where the '-' means that term does not exist. Hence the neutrino has no mass in the Standard Model. So neutrinos stay Weyl even after electroweak symmetry breaking. Of course we have discovered that neutrinos have mass thanks to neutrino oscillations. It's time to consider Neutrino Seesaw Masses. It was realized during the developments of grand unified theories that there often can be a right-handed neutrino. However, the right-handed neutrino was special because it has no charges under electroweak symmetry. This means that the right-handed neutrino can be different type of fermion known as a Majorana fermion which is still half of a normal fermion, but does not have a definite chirality like Weyl fermions. The right-handed neutrino can only be a Majorana particle because the Standard Model neutrinos have no electric charge. With the right-handed neutrino as a Majorana fermion, the mass matrix is different from all the other fermions of the Standard Model and is of the form

$$\mathcal{L}_{mass} = (\nu \quad \nu^c) \begin{pmatrix} 0 & \mathrm{m} \\ \mathrm{m} & \mathrm{M} \end{pmatrix} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} + h.c. \tag{141}$$

where M is some unknown mass scale. Now in most of physics, mass terms are as large as they are allowed to be. In grand unified theories, this turns out to be the grand unified scale, which is about a trillion times larger than the electroweak scale (which itself is about 100 times larger than the rest mass energy of the proton. What results is that the light eigenvalue is nearly completely the left-handed neutrino and is very light and the heavy eigenvalue is very heavy and is nearly purely the right-handed neutrino. The right-handed neutrino is far too heavy to be produced and thus we only observe a very light, left-handed neutrino. So this is the link between the neutrino being only left-handed, uncharged, and being very light. If we examine the lighter eigenvalue, which is the mass of the Standard Model neutrino, the mass is $m^2/M$, or about a trillion times smaller than other particles of the Standard Model, which are m. Most of the other fermions of the Standard Model are in the 0.1 GeV to 100 GeV range, this gives a neutrino mass in the 1 MeV to 100 MeV ranger, which is the ball park of what is observed (we do not know the absolute scale of neutrino masses, instead only known the differences of the mass squares of the neutrino masses)[7].

## 8.2 Diagonalization

Let us consider a matrix:

$$\begin{pmatrix} 0 & Ac\epsilon e^{\imath\alpha'} & As\epsilon e^{\imath\alpha'} \\ -Ac\epsilon e^{\imath\alpha} & B\epsilon s^2 & -B\epsilon sc \\ -As\epsilon e^{\imath\alpha} & -B\epsilon sc & B(1+\epsilon c^2) \end{pmatrix} \tag{142}$$

Here $\epsilon$ is a smallness parameter $\alpha$ and $\alpha'$ are phases. $c$ and $s$ these are angle sine and cosine. We have the hierarchy $B > A$. Without loss of generality we have only first horizontal-vertical line complex, other elements are real. This matrix can be brought to the diagonal form, however we have to use bi-unitary transformation:

$$U'^T Y U = Y_D = \begin{pmatrix} -Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_3 \end{pmatrix} \tag{143}$$

where we have some hierarchy too, like: $Y_3 \gg Y_2 \gg Y_1$. These are Yukawa eigenvakues for the physical fermions of three families. Now, firstly what we have to do is to remove these complex phases and then make diagonalization. For this we decompose unitary matrices $U = CO$ and $U' = C'O'$. Phase transformations have this form:

$$C' = \begin{pmatrix} e^{\imath(2\pi-\alpha')} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} e^{\imath(2\pi-\alpha)} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{144}$$

At the end of the day we have matrix without any phase and we can make orthogonal transformations,rotations, to make it diagonal.

$$Y = \begin{pmatrix} 0 & A\epsilon c & A\epsilon s \\ -A\epsilon c & B\epsilon s^2 & -B\epsilon sc \\ -A\epsilon s & -B\epsilon sc & B(1+\epsilon c^2) \end{pmatrix} \tag{145}$$

Consequently, we want-$O'^T Y O$, where O rotating matrix is:

$$O = O_{23}O_{13}O_{12} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & -s_{13} \\ 0 & 1 & 0 \\ s_{13} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{146}$$

The same is done for the prime part of orthogonal rotation. To compute these rotational angles we may use some interesting fact that $O^T Y^T Y O = O'^T Y Y^T O' = Y_D^2$. For the elements in our (145), approximation gives that:

$$\begin{aligned} B(1+\epsilon c^2) &\sim Y_3 \\ B\epsilon s^2 + \frac{B\epsilon^2 s^2 c^2}{(1+\epsilon c^2)} &\sim Y_2 \\ \frac{A^2 \epsilon c^2}{Bs^2} + \frac{A^2 \epsilon^2 s^2}{B(1+\epsilon c^2)} &\sim Y_1 \end{aligned} \tag{147}$$

We can see that we have second order smallness parameters inside, therefore, if we neglect it what remains is:

$$\begin{aligned} B(1+\epsilon c^2) &\sim Y_3 \\ B\epsilon s^2 &\sim Y_2 \\ \frac{A^2 \epsilon c^2}{Bs^2} &\sim Y_1 \end{aligned} \tag{148}$$

We start with the 23 rotation to diagonalize the lower 23 block:

$$O'^T_{23}Y^{(0)}O_{23} = \begin{pmatrix} 0 & Ac\epsilon c_{23} - As\epsilon s_{23} & Ac\epsilon s_{23} + As\epsilon c_{23} \\ -Ac\epsilon c'_{23} + As\epsilon s'_{23} & y_2 & 0 \\ -Ac\epsilon s'_{23} - As\epsilon c'_{23} & 0 & y_3 \end{pmatrix} \quad (149)$$

Angle that does this job is:

$$tan2\theta_{23} = \frac{2\epsilon sc(1+\epsilon)}{(\epsilon s^2)^2 - (1+\epsilon c^2)^2} = -\frac{\epsilon sin(2\alpha)}{1+\epsilon cos(2\alpha)} = tan2\theta'_{23} \quad (150)$$

As you can see diagonal terms are small $y$-s this is because there will be some corrections, that will be added and at the end of the day, we will get Yukawa coefficients. Now it is time for a next block (13) that will add small correction to the 11 and 33 elements, however they are way too small and we neglect it. However, 12 rotation implements sufficient correction to the 22 element that will be proportional to $Y_1$. Angles can be derived in the same way as it was in SeeSaw mechanism.

$$\begin{aligned} sin\theta_{13} &\sim -\frac{A\epsilon(cs_{23} + sc_{23})}{Y_3} \\ sin\theta'_{13} &\sim \frac{A\epsilon(cs_{23} + sc_{23})}{Y_3} \end{aligned} \quad (151)$$

and for the 12 rotation:

$$\begin{aligned} tan2\theta_{12} &\sim 2A\epsilon\frac{cc'_{23} - ss'_{23}}{Y_2 - Y_1} \\ tan2\theta'_{12} &\sim -2A\epsilon\frac{cc_{23} - ss_{23}}{Y_2 - Y_1} \end{aligned} \quad (152)$$

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