Geometric Methods in Mathematical Physics (28 lectures, 58 hours)

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1. **Elements of Set Theory**
   Overview of the course, Propositional and Predicate Logic, Zermelo-Frankel axioms, Classification of sets, Mappings between sets, Equivalence Relations

2. **Element of Topology-1**
   Topological Spaces, Convergence, Continuity, Homeomorphism

3. **Elements of Topology-2**
   Separation properties, Homotopic curves and Fundamental group

4. **Topological Manifolds**
   Topological manifolds, Bundles, Atlases

5. **Differentiable Manifolds**
   Differentiable manifolds, Diffeomorphism , Smooth manifolds

6. **Vector spaces**
   Vector spaces, Linear maps, Vector space isomorphism, Tensors, Algebraic fields, Modules

7. **Tangent and Cotangent Spaces**
   (Co)Tangent vector spaces to a manifold, Pull-back, Push-forward

8. **(Co)Tangent bundles**
   Tangent bundle, Vector fiber bundle, Sections and Vector fields, Lie derivative

9. **Differential Forms -1**
   n-forms, Wedge product, Exterior derivative, Hodge dual, Grassmann algebra

10. **Differential Forms-2**
    Gradient, Divergence, Curl, Stokes Theorem, De Rham cohomology, Betti numbers

11. **Differential Forms-3**
    Physical applications, Maxwell equations, Yang-Mills theory

12. **Principle Fiber Bundles**
    Lie groups, Principal fiber bundles, Associated bundles

13. **Kahler Manifolds**
    Symplectic Manifolds, Hermitian Manifolds, Kahler potential

14. **Complex Projective Spaces**
    C^N as a Kahler manifolds, CP^N as a Kahler manifold

15. **Hamiltonian Formalism-1**
    Integrability in classical mechanics, Hamiltonian vector fields, Killing potentials

16. **Hamiltonian Formalism-2**
    Lie algebra of Hamiltonian vector fields, Poisson brackets

17. **Hamiltonian Reduction**
    Reduction in case of involution and without involution

18. **Liouville–Arnold theorem**
    Liouville–Arnold theorem, action-angle variables, Conditionally periodic motion

19. **Harmonic Oscillator and Coulomb problem**
    Integrability in a quantum level, Symmetry algebra of harmonic oscillator, Fradkin tensor, Symmetry algebra of a Coulomb model, Laplace–Runge–Lenz vector

20. **Interaction with external magnetic Field**
    Inclusion of an external magnetic field in Hamiltonian mechanics, Landau problem

21. **Integrable models on Kahler manifolds**
    Kahler oscillator, C^N- and CP^N- oscillators and their hidden symmetries

22. **Quaternions**
    Algebra of quaternions, SU(2)-Instantons, Quaternionic spaces

23. **Hopf Maps**
    Zero, First and Second Hopf maps, Reduction examples via Hopf maps

24. **Example-1 (C^N -Smorodinsky-Winternitz system)**
    Real Smorodinsky-Winternitz system, Symmetry algebra, C^N- Smorodinsky-Winternitz and its symmetry algebra, quantization, U(1)-reduction

25. **Example-2 (CP^N-Rosochatius model)**
    CP^N-Rosochatius , conserved quantities, symmetry algebra, quantization

26. **Supersymmetry**
    Supersymmetry in d=4 Minkowski, SuperPoincare, Supermultiplets, Superspace , Superfield

27. **Supersymmetric Mechanics**
    Supersymmetric mechanics (N=2,N=4,N=8), d=1 SuperPoincare,

28. **SUSY on Kahler manifolds**
    SU(2|1)-SUSY algebra, SU(2|1) – Kahler superoscillaror and examples

References

1. T.Eguchi, P.B.Gilkey and A.J.Hanson, “Gravitation, gauge theories and differential geometry”, Physics Reports 66 (1980) 213-393
2. 2.M. Nakahara “Geometry, Topology and Physics”, Boca Raton, USA: Taylor & Francis, 2003
3. V. I. Arnold, “Mathematical methods of classical mechanics”, “Nauka” Publ., Moscow, 1989
4. A. Nersessian “Elements of (super-)Hamiltonian formalism” , Lect.Notes Phys.698:139-188,2006, [hep-th/0506170]