

INTEGRABILITY IN 2D STATISTICAL SYSTEMS AND IN 1+1 DIMENSIONAL QUANTUM FIELD THEORY.

THE FORM FACTOR PROGRAM

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56 hours, 28 lectures

1. INTEGRABLE SYSTEMS IN CLASSICAL AND QUANTUM MECHANICS (4 hours).

Hamilton-Jacobi equation, The Integrals of motion , The Liouville's theorem, Canonical variables, Chaos in dynamical systems, KAM theorem, N-body problem. Toy examples of classical and quantum integrable models.

2. THE YANG FUNDAMENTAL EXAMPLE-FIRST EXACT TWO PARTICLE S-MATRIX (2-hours).

1d N-fermion (with spin) delta function interaction Hamiltonian. Exact diagonalization and exact two particle S-matrix. Yang functional equation for S-matrix. N-particle S matrix is $N(N-1)/2$ ordered product of the 2 particle S-matrices.

3. THE TENSOR PRODUCT OF THE MATRICES AND VERIFICATION OF THE YANG EQUATION USING ONLY SPACE TENSOR PRODUCT (2 hours).

Tensor product of the matrices . Permutation operator .

4. VERTEX MODELS IN 2D LATTICE.THE 8-VERTEX MODEL (2 hours).

The Boltzman vertex weights matrix. The transfer matrix. Commutativity of the transfer matrix and integrals of motion. The fundamental equation for the vertex weights . Baxter equation (2 hours).

5. THE ELLIPTIC FUNCTIONS AND SOLUTION OF THE BAXTER EQUATION (2 hours).

The elliptic functions. Three linear Baxter functional equations as a algebraic differential equation for the vertex weights. The elliptic parametrization of the vertex weights.

6. TRIGONOMETRIC AND RATIONAL LIMIT OF THE 8 -VERTEX MODEL (2 hours).

Trigonometric and rational limit of the elliptic function and 6 vertex model . Yang fundamental S-matrix and Baxter rational solution. The Yang-Baxter equation.

7. BELAVIN-DRINFELD THEOREM AND CLASSIFICATION OF SOLUTION YANG-BAXTER EQUATION (2 hours).

The quasiclassical parameter of the R matrix and limit of the Yang-Baxter equation. Rational, trigonometric and elliptic solution of the classical Yang-Baxter equation and root system. The classification solution of the quantum Yang-Baxter equation.

8. THE QUANTUM GROUP AND YANG-BAXTER EQUATION (2hours).

The quantum deformation of the Lie algebras. Co-product and antipod. The representation of the $SL_q(2)$ group, solution of the Yang-Baxter equation and quantum group.

9. THE EXACT INTEGRABILITY IN 1+1 DIMENSIONAL QUANTUM FIELD THEORIES (2 hours).

The conservation laws and multiparticle production. Factorization of N particle S-matrix on the $N(N-1)/2$ two particle S matrices.

10. THE BOOTSTRAP PROGRAM IN 1+1 DIMENSIONAL EXACT INTEGRABLE QUANTUM FIELD THEORIES (2hours).

Unitarity, Crossing relation, Yang Baxter equation. $O(N)$ symmetric solution of the Yang-Baxter equation and nonlinear sigma and Gross-Neveu models.

11. THE ALGEBRAIC BETHE ANSATZ FOR THE XXX $\frac{1}{2}$ -HEISENBERG CHAIN (2 hours).

The R-matrix and L-operator. Creation and annihilation operators. The monodromy operator and Transfer matrix. The construction of the Bethe wave vector. The Bethe Ansatz equation.

12. THE FUSSION OF THE R-MATRICES AND HIGHER SPIN HEISENBERG CHAINS (4 hours).

Non-symmetric solutions of the Yang-Baxter equation-the spaces with different representation. The family of the commuting transfer matrices. Higher spin Heisenberg chains. Exact diagonalization Higher spin chains, XXX and XXZ higher spin chains.

13. THE BETHE ANSATZ EQUATIONS AND ITS SOLUTIONS (2 hours).

The H.Bethe string hypothesis in thermodynamic limit. The density functions on the center of strings. Bethe ansatz equations for XXX and XXZ Heisenberg chains and strings.

14. THE YANG-YANG THERMODYNAMICS AND THE SYSTEM OF INTEGRAL EQUATIONS (4-hours).

Thermodynamic limit of the Bethe Ansatz equation. The definition of the entropy. Investigation of the integral equations at the $T \ll 1$, $H=0$ - the specific heat $C(T)$. And free energy at $H \ll 1, T=0$. The Winer-Hopt method solution of the integral equation on half line.

15. EXACT SOLUTION OF THE XYZ OR 8-VERTEX MODEL (4-hours).

The 8-vertex model on the lattice. The gauge transformation of the L operator. Faddeev –Takhtajan Generalized Bethe Ansatz.

16. THE COMPLETENESS OF THE STATES IN THE ALGEBRAIC BETHE ANSATZ (2-hours)

The counting of all Bethe states in XXX Heisenberg model using string hypothesis.

17. THE LOW-LIENGE EXCITATIONS IN THE XXX HEISENBERG MAGNET (2 hours).

18. THE FORM FACTOR PROGRAM (2-hours).

The bootstrap program in 1+1 dimensional integrable QFT. Exact S-matrix, Crossing relation, Unitarity, the exact form factors, the correlation functions.

19. THE KAROWSKI-WEISZ-SMIRNOV EQUATION (2-hours).

For the matrix element of local operator (between exact vacuum and asymptotic state) one has formulated the system of functional equations.

20. THE MASTER FORMULA-THE GENERAL SOLUTION OF THE FORM FACTOR EQUATIONS (2-hours).

Using the so-called generalized or off-shell Bethe ansatz one can construct the integral representation, for exact form factors. This master formula is correct for all exact integrable QFT's. The notion of the P-function for different operators.

21. THE SIMPLE DIAGONAL QFT- S=-1 ISING FIELD THEORY (2-hours).

The simplest example for form-factor program-Ising field theory. Application of the master formula in the case S=-1. The p-function for energy and spin field.

22. THE SINH-GORDON MODEL-EXACT FORM FACTOR FOR THE FIELD AND EXPONENTIAL FIELD (2 hours).

The P-function for the field and exponential field.

23. EXACT QUANTUM EQUATION OF MOTION IN SINH-GORDON QUANTUM FIELD THEORY (2 hours).

By appropriate normalization of the field exponential field form factors we get exact sine-Gordon quantum equation of motion.

24. THE FORM FACTORS FOR THE Z(N) ISING MODEL and A(N) TODA FIELD THEORY (2 hours).

Using exact S-matrix for Z(N) Ising model and the master formula we construct form factors for the infinity many operators. The same we did for Toda field theory.

25. THE CONCLUSION AND REMARKS. THE PROBLEMS FOR FUTURE INVESTIGATION.

REFERENCES

- 1.R.J.BAXTER, Exactly Solved Models in Statistical Mechanics(Academic,London,1982)
- 2.L.D.Faddeev, L. A. Takhtajan , Usp..Mat.Nauk.34 ,13(1979)
- 3.A.A.Belavin, V.G. Drinfeld , Func. Anal. Appl. 16,159(1982)
- 4.A.B.Zamolodchikov, Al .B . Zamolodchikov, Annals of Physics 120,253-291(1979)
- 5.M.Takahashi, Thermodynamics of one-dimensional solvable models,
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