Syllabus of the Course

Course Title	**Modern Trends in Mathematical Physics II**
Author(s)	**Grigori Giorgadze**, PhD
Curse Code	
Status of the Course	Faculty: **Exact and Natural Sciences** Study Level: **PhD** Program: **Physics, Elementary Particle Physics** Status: **Compulsory Elective**
ECTS Credits	Credits: **5 ECTS** Total number of hours: **125 hours** Contact hours: **45 hours** Lectures: **1 hours per week (in total 15 hours)** Seminars: **2 hours per week (in total 30 hours)** Practical classes: Working group: Laboratories: Etc. Non-contact hours: **65 hours**
Lecturer(s)	Name, Surname: **Grigori Giorgadze** Status: **Professor** Employment: **Javakhishvili Tbilisi State University** Mobile: **+ 995 555 73 11 77** E-mail: **gia.giorgadze@tsu.ge**
Course Objectives	The aim of the course is to provide students with the basic knowledge on the mathematical methods used in different branches of physics. The first 8 lectures cover different aspects of the Hilbert spaces and linear operators on unitary spaces, symplectic geometry and Newton's, Lagrange's and Hamilton's formulation of classical mechanics. In the second part of the course (last 7 lectures) we consider different aspect of variational problems in pseudo-Riemanian and sub-Riemanian geometry.
Pre-conditions	Basic calculus, Linear algebra
Main Results of the course	**Knowledge and Understanding:** The graduate: • will be aware of mathematical methods used in descriptions of physical models; • will have skills of analytic and numerical calculus; • will be able to participate in discussions with experts in the field. **Skills:** The graduate will be able: • practically develop the methods of solution of various physical problems; • to gather information and conduct independent research; • to create new physical ideas and discuss them with the colleagues. • to explore new tasks, using various mathematical and physical methods. **Responsibility and Autonomy:** • The graduate can conduct innovative research in the field.

Methods of Teaching/Study	Main methods of teaching are lectures and seminars, interactive discussions, written and home tasks, team work, deductive and analytical methods
Evaluation Criteria	**The forms of evaluation and points:** Attendance/activity **10 points** Seminar (reports + activity) **30 point** Colloquium (written + oral) **20 point** Final exam: **40 points** **Pre-condition for final exam:** The minimum threshold competence of interim evaluation is **11 points**. **Criteria of evaluation:** Maximum positive evaluation - **100 points;** Minimum positive evaluation **- 51 points** During the process of study a student's knowledge is evaluated according to: participation in discussions at lectures and seminars, active engagement at seminars, performance of practical and written assignments, oral presentation, answering questions, preparation of course works. Evaluation system envisages 5 types of positiv and 2 types of negative evaluations: 91 - 100 points (A) ---Excellent 81 - 90 points (B) -----Very good 71 - 80 points (C) -----Good 61 - 70 points (D) -----Satisfactory 51 - 60 points (E) -----Sufficient 41 - 50 points (Fx) ----Did not pass (Student needs to work harder to pass the examination and is allowed to take an additional exam after working independently). 0 – 40 points (F) ------Failed (The work accomplished by the student is not sufficient and he/she must take a course anew).
Basic Literature	**Literature** (Can be found in TSU libraries)**:** [1] A.Kostrikin, Yu.Manin. Lienear algebra and Geometry, Gordon and Breach Science Publ.1997 [2] I.Agricola, Th.Friedrich. Global analysis.AMS,2000 [3] M. Puta. Hamiltonian mechanical systems and Geometric quantization. 1994 [4] M.de Gooson, Symplectic geometry and quantum mechanics, Birkhauser verlag, 2006 [5] A. Cannas da Silva, Lecture on symplectic geometry, Springer, 2008 [6] A.Agrachev, D.Barilari, U.Boscain. A Comprehensive Introduction to Sub-Riemannian Geometry, Cambridge Uni.Press. 2020 [7] O. Calin, D.-C. Chang. Sub-Riemannian Geometry: General Theory and Examples. Cambridge Uni.Press. 2009.

Supplementary Materials	
Additional Information/Conditions	Administration will provide consultations for students

Content of the Course

1.	**Bilinear forms on functional spaces. Geometry of functional spaces with weighs** (trigonometric, Legendre, Chebyshev, Hermite plyninomiasl and their orthogonalization)	[1] Chapters 2, section 3,4
2.	**Unitary space, complexification and polarization** (invariant antisymmetric scalar product respect to multiply complex unity, state space of quantum system, Feynman rule)	[1] Chapter 2, section 6 [5] part 1, pp.3--11
3.	**Symplectic space and two dimensional symplectic geometry** (characteristic polynomials of symplectic matrix, Pfaffian, relations between orhtogonal, symplectic and unitary groups)	[1] Chapter 2, section 13 [4] section 2, pp. 27-38
4.	**Physical interpretation of the Minkowski space** (triangle inequality, Lorentz transformation and multiplier, orientations)	[1] Chapter 2, section 12
5.	**Selfadjoint operators in quantum mechanics (**Heisenberg's uncertainty principle, energy spectrum and stationary states)	[1] Chapter 2 Section 9 [4] section 8, pp. 278-266
6.	**Formulation of mechanics according to Newton** (autonomous Newtoian system, Newtonian system with potential energy, energy conservation for Newtonian system, Maupertuis-Jacobi principle)	[2] Chapter 7, pp. 253-257;exercises pp.264-269
7.	**Formulation of mechanics according to Lagrange** (Lagrange function, d'Alambert-Lagrange theorem, Lagrangian system, pseudo-Riemannian manifold, action integral, principle of last action, Legendre transformation, conservation of energy for Lagrangian systems, Noether's theorem)	[2] Chapter 7, pp. 257- 262; exercises pp.264-269 [4] section 5, pp.123-136
8.	**Formulation of mechanics according to Hamilton** (Hamiltonian, Hamilton's theorem)	[1] section 2.1; [2] Chapter 7, pp. 262-264; exercises pp.264-269 [4] section 2, pp. 50-71 [5] part 7, pp.127-135
	Colloquium	

9.	**Lie-Poisson structure on symplectic manifold.** (Poisson bracket, Poisson geometry, local structure of Poisson manifolds)	[3] Chapter 2; Problems and solutions pp.44-51
10.	**The Laplasian on differential forms** (adjoint operator, adjoint operator in pseudo-Riemannian manifold, Hodge-Laplase operator, harmonic pseudo-Riemannian manifold)	[2] Chapter 3; Problems and solutions 105-110
11.	**Differential forms in thermodynamics** (Statistical states of Hamiltonian system, fundamental theorems of thermodynamics, thermodynamical interpretation of horizontal connectivity, Caratheodory's Theorem)	[2] Chapter 8; Problems and solutions pp.292--294; [7] chapter 3, pp.73-75
12.	**Geometry of surface in 3D-space** (Parallel transport, Gauss–Bonnet theorems, Minkowski inner product, Spaces of constant curvature)	[6] chapters 2 pp.19-40, examples pp.41-44
13.	**Sub-Riemannian manifold** (The existence of the sub-Riemannian metric, orthogonal vector field at point, horizontal gradient, connection and curvature forms)	[7] Chapter 2; pp. 37-61 Examples and application pp. 175-229
14.	**Lagrangian and Hamiltonian formalism on sub-Riemannian manifolds** (Horizontal and flat connection Euler-Lagrange equations)	[7] chapter 5 and 6; Examples 231-263
15.	**The Sub-Riemannian Heat Equation** (Heat equation in Riemannian and sub-Riemannian context, Heat Equation on the Heisenberg Group, fundamental solution)	[6] chapter 21,pp. 674-678, pp.686-688.
16.	**Exam**	
17.	**Re-examine**	