Modern Trends in Mathematical Physics II

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45 hours: 15 lectures (in total 15 hours), 15 Seminars (in total 30 hours)

1.	**Bilinear forms on functional spaces. Geometry of functional spaces with weighs** (trigonometric, Legendre, Chebyshev, Hermite plyninomiasl and their orthogonalization)	[1] Chapter 2, section 4
2.	**Unitary space, complexification and polarization** (invariant antisymmetric scalar product respect to multiply complex unity, state space of quantum system, Feynman rule)	[1] Chapter 2, section 6; [6] part 1, pp.3--11
3.	**Symplectic space and two dimensional symplectic geometry** (characteristic polynomials of symplectic matrix, Pfaffian, relations between orhtogonal, symplectic and unitary groups)	[1] Chapter 2, section 13; [5] section 2, pp. 27-38
4.	**Physical interpretation of the Minkowski space** (triangle inequality, Lorentz transformation and multiplier, orientations)	[1] Chapter 2, section 12
5.	**Selfadjoint operators in quantum mechanics** (Heisenberg's uncertainty principle, energy spectrum and stationary states)	[1] Chapter 2 Section 9; [5] section 8, pp. 278-266
6.	**Formulation of mechanics according to Newton** (autonomous Newtoian system, Newtonian system with potential energy, energy conservation for Newtonian system, Maupertuis-Jacobi principle)	[2] Chapter 7, pp. 253-257; exercises pp.264-269
7.	**Formulation of mechanics according to Lagrange** (Lagrange function, d'Alambert-Lagrange theorem, Lagrangian system, pseudo-Riemannian manifold, action integral, principle of last action, Legendre transformation, conservation of energy for Lagrangian systems, Noether's theorem)	[2] Chapter 7, pp. 257- 262; exercises pp.264-269; [5] section 5, pp.123-136
8.	**Formulation of mechanics according to Hamilton** (Hamiltonian, Hamilton's theorem)	[1] section 2.1; [2] Chapter 7, pp. 262-264; exercises pp.264-269

		[5] section 2, pp. 50-71; [6] part 7, pp.127-135
	Colloquium	
9.	**Lie-Poisson structure on symplectic manifold** (Poisson bracket, Poisson geometry, local structure of Poisson manifolds)	[3] Chapter 2; Problems and solutions pp.44-51
10.	**Quantitation** (Quantization schemes, general description of the canonical, geometric and deformation quantizations)	[3] Chapter 1
11.	**Geometric prequantization** (quantization and Dirac problem, complex line bundle and Dirac problem, Souriau's theorem)	[3] Chapter 6; Problems and solutions pp.168-181
12.	**Polarizations and the first attempts to quantization** (integrable distributions on symplectic manifolds, foliation)	[3], Chapter 7; Problems and solutions pp.199-207
13.	**Sufficiently condition of quantization** (prequantizations of canonical transformation, Bohr-Sommerfeld condition, polarization and quantization)	[4] chapters 8 and 9; [6] part 6, pp.101-115
14.	**Path integral and Feynmann expansion (**WKB approximation, pairing, relativistic quantization, holomorphic quantization, projections and Fock spaces)	[4] chapter 9 pp.171-217
15.	**Metaplectic correction** (corrected quantization, metaplectic representation, parallel transform, nonnegative Lagrangian subspaces, real Lagrangian subspaces, corrected Bohr-Sommerfel condition)	[4] chapter 10; [5] section 7, pp.196-227; [6] part 7, 143-147
16.	**Exam**	
17.	**Re-examine**	

Literature:

[1] A. Kostrikin, Yu. Manin. Lienear algebra and Geometry, Gordon and Breach Science Publ.1997

[2] I. Agricola, Th. Friedrich. Global analysis.AMS,2000

[3] M. Puta. Hamiltonian mechanical systems and Geometric quantization. 1994

[4] N. Woodhouse, Geometric quantization, second edition, Oxord Uni., 1997

[5] M.de Gooson, Symplectic geometry and quantum mechanics, Birkhauser verlag, 2006

[6] A. Cannas da Silva, Lecture on symplectic geometry, Springer, 2008