Modern Trends in Mathematical Physics

Gia Giorgadze
Iv. Javakishvili Tbilisi State University

60 hours: 15 lectures (in total 30 hours), 15 Seminars (in total 30 hours)

1.	**Vector space, space of linear functions, fields of complex numbers and generalizations** (dual vector space, hyperbolic numbers, quaternions, octonions, spinors, group of motions, matrices of geometric numbers, special relativity and Lorencial geometry).	[1] Chapters 1-3
2.	**Differentiable manifold, tangent and cotangent space** (vector fields, complex of differential forms, duality, integration of differential forms, curves and surfaces)	[2] Chapter 5
3.	**Complex structure on manifolds (**Riemann surface, differential and integral calculus on Riemann surfaces)	[2] Chapter 4, Chapter 11
4.	**Systems of ordinary differential equations on complex plane and Riemann sphere I** (regular and Fuchsian systems, boundary value problems, monodromy representation)	[5] Chapter 2
5.	**Systems of ordinary differential equations on complex plane and Riemann sphere II** (Isomonodromic deformation, nonlinar diffrential equations in isomonodromic problems)	[5] Chapter 5
6.	**Vector bundle on manifolds, tangent and cotangent bundle** (coordinate and invariant descriptions, duality, Hopf fibration, description of holomorphic vector bundles on Riemann sphere, space of holomorphic and meromorphic sections)	[2] Chapter 2
7.	**Connection and curvature on vector bundles** (holomorphic vector bundles induced from Fuchsian system, connection 1-form, solutions space of Fuchsian systems as sections of holomorphic vector bunles with connection)	[2] Chapter 10
8.	**Rank 2 holomorphic vector bundle on Riemann sphere** (Riemann and Hypergeometric equations, hypergeometric functions and related topics including properties of special functions)	[5] Chapter 3, Apendix 2

	Colloquium	
9.	**Orthogonal polynomials** (Orthogonal polynomials as solutions special type second order Fuchian equations, analytic extension, electrostatic interpretation of zeros of orthogonal polynomials)	[5] Chapter 2
10.	**Nonlinear PDE** (Inverse scattering problem, Painleve transcendent, KdV, nonlinear Schrodinger and sin-Gordon equations)	[4] Chapter 1 and Chapter 4; [5]
11.	**Multimidensonal calculus of variations** (variational derivative, Euler-Lagrange equations, energy-momentum tensor and conservation laws, electromagnetic field equation, Dirichlet functional and harmonic mapping)	[2] Chapter 14
12.	**Distributions (**Physical approach: problem of distribution of charge, problem of momentum; regular and singular distributions, derivation of distributions, convolution, physical interpretation of convolution operators, Cauchy principal value)	[3], Chapters 7-8.
13.	**Fourier analysis** (Hilbert space, Fourier series, Fourier transform functions and distributions, Sturm-Liouville problem, Gibbs phenomenon)	[3] Chapter 9-11
14.	**Geometric fields in physics I** (gravitation field as metric, action functional of gravitational field, Schwarzschild and Kerr metrics, interaction of matter with gravitational field)	[2], Chapter 15
15.	**Geometric fields in physics II (**Dirac equation, Yang-Mils field and equations)	[2], Chapter 15
	Exam	

Literature:

[1] G. Sobczyk. Matrix Gateway to Geometric Algebra, Spacetime and Spinors, 2019

[2] S.Novikov, I.Taimanov. Modern geometric structures and fields, 2006

[3] W.Appel, Mathematics for physics and physicists, Princeton Uni.Press, 2007

[4] M.J. Ablowitz and H. Segur, Solitons and the Inverse Scattering Transform. SIAM, Philadelphia, 1981

[5] A.Fikas, A. Its, A. Kapaev, V Novokshenov. Painleve transcendents: The Riemann-Hilbert approach, 2006