Bounds on Supersymmetric Operators from Experiments

Archil Suladze

Masterarbeit in Physik
angefertigt im Physikalischen Institut

vorgelegt der
Mathematisch-Naturwissenschaftlichen Fakultät
der
Rheinischen Friedrich-Wilhelms-Universität
Bonn

July 2020

I hereby declare that this thesis was formulated by myself and that no sources or tools other than those cited were used.

Bonn, 03.07.2020 ....

Date Signature

1. Gutachter: Prof. Dr. Herbert Dreiner
2. Gutachter: PD. Dr. Akaki Rusetsky

Contents

1 Introduction

The Standard Model (SM) of particle physics is an extremely successful theory providing various predictions that have been thoroughly tested and approved by experiments. Despite the brilliance of the Standard Model, it is still far from being a complete theory. There are number of unsolved problems that this theory is unable to provide solutions for. In order to discover physics beyond the Standard Model different new theories and extensions have been proposed. Among them are the supersymmetric theories that are considered to be an elegant solution to some of the current problems in particle physics. Supersymmetry is a spacetime symmetry relating bosons and fermions to each other and easily remedies the hierarchy problem. In my thesis I will mainly focus on the minimal phenomenologically viable supersymmetric extension of the SM, called the Minimal Supersymmetric Standard Model (MSSM). The conservation of baryon number (B) and lepton number (L) present in the Standard Model is no longer guaranteed in supersymmetric models. For instance, the MSSM Lagrangian can generally contain additional terms, both gauge-invariant and renormalizable, that violate either B or L [2]. A new symmetry, called R-parity, can be imposed to forbid such terms and this way make sure to stay close to current experimental picture. Nevertheless, the presence of R-parity violating ($\not{R}_p$) couplings in the supersymmetric Lagrangians leads to new types of interactions. Some of these interactions yield extra contributions to the well-known SM processes, while others generate reactions that are forbidden in the Standard Model. In contrast to the SM, where all the processes happen via the exchange of fundamental spin-1 gauge bosons, new fundamental scalar bosons can be additionally mediated between SM fermions in the MSSM if R-parity is violated. Therefore, R-parity violation, if present at all, leads to many interesting new phenomenological consequences. The non-observation of certain interactions and measured experimental constraints on the observables set stringent bounds on the $\not{R}_p$ coupling constants. The main goal of my thesis is to recalculate some of these bounds based on the updated experimental values for different observables provided by the Particle Data Group.

One thing to note is that it is generally more convenient to use two-component Weyl spinor notation for fermions when working in electroweak or in supersymmetric theories. As some readers may not be familiar with it, I will briefly provide the basic formalism before specifying the MSSM Lagrangian and calculating different matrix elements or observables. Section 2 covers the essential notations, conventions and Feynman rules in two-component language heavily based on Ref. [1]. In section 3, using the formalism of section 2, I shortly present the interaction Lagrangian of the SM in two-component notation together with the associated Feynman interaction vertices. Section 4 is dedicated to a brief overview of supersymmetry and the MSSM based of Ref. [2]. In section 5 I discuss R-parity, its origin, properties, violation and consequences based on Ref.s [3] and [2]. In the end of this section

I specify the $\not{R}_P$ trilinear interaction terms. In Section 6, I go through various low energy processes to calculate $\not{R}_P$ contributions coming from the above-mentioned trilinear terms and, consequently, set bounds on the trilinear $\not{R}_P$ coupling constants involved. Finally, in the last section, I will summarize all the obtained single-coupling bounds in a table together with previous bounds found in the literature.

2 Two-component spinors

One of the distinctive features of the Standard Model is the chiral nature of fermionic interactions. The fundamental degrees of freedom for fermions are two-component Weyl-van der Waerden spinors that transform under the irreducible representations of the Lorentz group [1]. It can be shown that the Lie algebra for the Lorentz group splits into two commuting $su(2)$ subalgebras [5]:

$$so(1,3) = su(2) \oplus su(2). \tag{2.1}$$

Each irrep (short for irreducible representation) of $su(2)$ is characterized by a number j that is either integer or half-integer. This in turn means that we can specify the irreps of the Lorentz group istelf by two such numbers, say j_1 and j_2, generating infinite number of irreducible representations denoted by (j_1, j_2) with dimension $(2j_1+1)(2j_2+1)$. The two relevant irreps are $\left(\frac{1}{2},0\right)$ and $\left(0,\frac{1}{2}\right)$, referred to as spin-$\frac{1}{2}$ representations. The elements living in the vector spaces on which these irreps act are known as Weyl spinors. The $\left(\frac{1}{2},0\right)$ & $\left(0,\frac{1}{2}\right)$ spinors are called the left-handed and right-handed Weyl spinors, respectively, and are usually denoted by ψ_L & ψ_R in the familiar four-component notation. One should keep in mind that these irreps are indeed distinct, though related by Hermitian conjugation, as the left- and right-handed Weyl spinors tranform differently under boosts and rotations [5]. The chiral nature lies in the fact that the two kinds of Weyl spinors are also differently charged under the Standard Model gauge group. The parity is thus violated by the SM Lagrangian. In the four-component notation, the Dirac fermion, containing both left-and righ-handed Weyl degrees of freedom, therefore has to carry left- or right-handed projection operators:

$$P_L \coloneqq \frac{1}{2}(\mathbb{1}-\gamma_5), \qquad P_R \coloneqq \frac{1}{2}(\mathbb{1}+\gamma_5) \tag{2.2}$$

all along. So, while the parity-conserving theories, such as QCD and QED, are well-suited to the four-component Dirac spinor notation, the latter may become a bit clumsy in a theory that violates parity. Moreover, the two-component notation has even more advantage in supersymmetric models, since their building blocks are chiral and vector supermultiplets, both of which contain a sole two-component Weyl fermion.

2.1 Conventions, notations and two-component spinor identities

This subsection specifies the most essential conventions and notations. I will also present some of the two-comonent spinor identities that are relevant for the later calculations. The possible number of these identities is enormous, so the reader is encouraged to refer to section 2 of Ref. [1] and the references in it for more detailed discussion of the two-component spinor notation. All conventions, notations and identities are taken from Ref. [1].

I will use the following metric tensor:

$$g_{\mu\nu} = g^{\mu\nu} = \mathrm{diag}(+1, -1, -1, -1)\,, \tag{2.3}$$

where $\mu, \nu = 0, 1, 2, 3$ are spacetime vector indices.

A two-component $\left(\frac{1}{2}, 0\right)$ spinor field is denoted by ψ_α. It is a two-component, complex, anti-commuting field with $\alpha = 1, 2$. A two-component $\left(0, \frac{1}{2}\right)$ spinor is denoted by $\psi^\dagger_{\dot\alpha}$ with $\dot\alpha = 1, 2$ as well[1]. It is important to distinguish between the undotted and dotted spinor indices because they cannot be directly contracted to form Lorentz invariant quantities. As mentioned above, the $\left(\frac{1}{2}, 0\right)$ and $\left(0, \frac{1}{2}\right)$ representations are related by hermitian conjugation. That is, if ψ_α is a $\left(\frac{1}{2}, 0\right)$ fermion, then $\psi^\dagger_{\dot\alpha}$ transforms as a $\left(0, \frac{1}{2}\right)$ fermion [1]:

$$\psi^\dagger_{\dot\alpha} \equiv (\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha}, \tag{2.4}$$

and conversely

$$(\psi^\dagger_{\dot\alpha})^\dagger = \psi_\alpha\,. \tag{2.5}$$

This implies that one can use left-handed spinor fields ψ_α or right-handed fields $\psi^\dagger_{\dot\alpha}$ only to describe all fermion degrees of freedom. By standard convention the $\left(\frac{1}{2}, 0\right)$ Weyl spinors are chosen and I will also stick to this choice later on when defining the chiral supermultiplets of the MSSM. Note that it is helpful to regard ψ_α as a column vector, and $\psi^\dagger_{\dot\alpha}$ as a row vector.

There are also spinors that have raised spinor indices and are denoted as ψ^α and $\psi^{\dagger\dot\alpha}$. One can picture ψ^α as a row vector, and $\psi^{\dagger\dot\alpha}$ as a column vector when combining them to form Lorentz tensors [1]:

$$\psi^{\dagger\dot\alpha} \equiv (\psi^\alpha)^\dagger = (\psi^\dagger)^{\dot\alpha}. \tag{2.6}$$

The height of spinor indices matters. In order to raise or lower them one can introduce the analogue of metric tensor for spinors, the two-index antisymmetric epsilon symbol defined as [1]

$$\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1\,, \qquad \epsilon^{11} = \epsilon^{22} = \epsilon_{11} = \epsilon_{22} = 0\,. \tag{2.7}$$

[1]spinor indices are conventionally denoted by the symbols from the beginning of the Greek alphabet.

We can then formally define $\epsilon^{\dot{\alpha}\dot{\beta}} \equiv (\epsilon^{\alpha\beta})^*$ and $\epsilon_{\dot{\alpha}\dot{\beta}} \equiv (\epsilon_{\alpha\beta})^*$ [1]. Viewing ϵ as a 2×2 matrix yields the following [1]:

$$\psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta, \qquad \psi_\alpha = \epsilon_{\alpha\beta}\psi^\beta, \qquad \psi^{\dagger\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\psi^\dagger_{\dot{\beta}}, \qquad \psi^\dagger_{\dot{\alpha}} = \epsilon_{\dot{\alpha}\dot{\beta}}\psi^{\dagger\dot{\beta}}. \tag{2.8}$$

In order to obtain Lorentz vectors hermitian sigma matrices $\sigma^\mu_{\alpha\dot{\beta}}$ and $\overline{\sigma}^{\mu\,\dot{\alpha}\beta}$ are introduced as follows [1]:

$$\sigma^\mu = (\mathbb{1}_{2\times 2}\,;\,\vec{\boldsymbol{\sigma}})\,, \qquad \overline{\sigma}^\mu = (\mathbb{1}_{2\times 2}\,;\,-\vec{\boldsymbol{\sigma}})\,, \tag{2.9}$$

where $\vec{\boldsymbol{\sigma}} \equiv (\sigma^1\,,\,\sigma^2\,,\,\sigma^3)$ represents the three-vector of Pauli matrices. Using space-time metric tensor the covariant versions can be easily obtained [1]:

$$\sigma_\mu = g_{\mu\nu}\sigma^\nu = (\mathbb{1}_{2\times 2}\,;\,-\vec{\boldsymbol{\sigma}})\,, \qquad \overline{\sigma}_\mu = g_{\mu\nu}\overline{\sigma}^\nu = (\mathbb{1}_{2\times 2}\,;\,\vec{\boldsymbol{\sigma}})\,. \tag{2.10}$$

When constructing Lorentz tensors lowered indices must only be contracted with raised indices and vice versa. Following a convention, descending contracted undotted indices and ascending contracted dotted indices,

$$^{\alpha}{}_{\alpha} \qquad \text{and} \qquad {}_{\dot{\alpha}}{}^{\dot{\alpha}} \tag{2.11}$$

can be suppressed [1].

Until now I have been discussing *anticommuting* fermion quantum fields. In the later sections *commuting* spinor wave functions will also appear, for example from the Feynman rules. Consequently, some identities will generate a relative minus sign depending on the type of spinors involved. Therefore, it is convenient to denote the generic spinor by z_i [1], where i enumerates different spinors. The extra minus sign, when interchanging the order of two anticommuting fermion fields, can be incorporated in a handy notation [1]:

$$(-1)^A \equiv \begin{cases} +1\,, & \text{commuting spinors,} \\ -1\,, & \text{anticommuting spinors.} \end{cases} \tag{2.12}$$

Using eq. (2.12) the following identities involving z_i hold[2] [1]:

$$z_1 z_2 = -(-1)^A z_2 z_1\,, \tag{2.13}$$

$$z_1^\dagger z_2^\dagger = -(-1)^A z_2^\dagger z_1^\dagger\,, \tag{2.14}$$

$$z_1 \sigma^\mu z_2^\dagger = (-1)^A z_2^\dagger \overline{\sigma}^\mu z_1\,. \tag{2.15}$$

[2]The additional minus sign in equations (2.13) and (2.14) appears due to the antisymmetry of the ϵ symbol. Because of this, interchanging commuting spinors results in an overall minus sign, while interchanging anticommuting spinors does not produce an extra minus sign.

Often one needs to simplify or rearrange Two-component spinor products, for example in the matrix element calculations. This can be achieved by the Fierz identities. Here I list some of the relevant ones used later in the matrix element calculations [1]:

$$(z_1\sigma^\mu z_2^\dagger)(z_3^\dagger\overline{\sigma}_\mu z_4) = 2(z_1 z_4)(z_2^\dagger z_3^\dagger)\,, \tag{2.16}$$

$$(z_1^\dagger\overline{\sigma}^\mu z_2)(z_3^\dagger\overline{\sigma}_\mu z_4) = 2(z_1^\dagger z_3^\dagger)(z_4 z_2)\,, \tag{2.17}$$

$$(z_1\sigma^\mu z_2^\dagger)(z_3\sigma_\mu z_4^\dagger) = 2(z_1 z_3)(z_4^\dagger z_2^\dagger)\,. \tag{2.18}$$

A more comprehensive list of sigma matrix identities and their associated Fierz identities are given in Appendix B of Ref. [1].

2.2 Correspondence to four-component spinor notation

Before discussing Feynman rules, let me briefly demonstrate the correspondence between the two-component spinor notation and the more familiar four-component Dirac spinor notation. Of course the two notations should be linked to each other and the results obtained from the calculations should not depend on which notation one employs. The easiest way to make the correspondence evident is to go to the Chiral representation in which γ-matrices take the following form[3] [1]:

$$\gamma^\mu \equiv \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot\beta} \\ \overline{\sigma}^{\mu\,\dot\alpha\beta} & 0 \end{pmatrix}\,, \quad \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\delta_\alpha{}^\beta & 0 \\ 0 & \delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}\,, \quad \mathbb{1} = \begin{pmatrix} \delta_\alpha{}^\beta & 0 \\ 0 & \delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}\,, \tag{2.19}$$

where the 4×4 identity matrix is also displayed in terms of dotted and undotted Kronecker symbols.

In a Chiral representation, a Dirac spinor is obtained by combining mass-degenerate left- and right-handed two-component Weyl spinors, say χ_α and $\eta^{\dagger\dot\alpha}$, of opposite $U(1)$ charge into a single four-component object [1]:

$$\Psi(x) \equiv \begin{pmatrix} \chi_\alpha(x) \\ \eta^{\dagger\dot\alpha}(x) \end{pmatrix}\,, \tag{2.20}$$

while the Dirac conjugate field in the same representation is

$$\overline{\Psi}(x) \equiv \Psi^\dagger(x) \begin{pmatrix} 0 & \delta^{\dot\alpha}{}_{\dot\beta} \\ \delta_\alpha{}^\beta & 0 \end{pmatrix} = \begin{pmatrix} \eta^\alpha(x), & \chi^\dagger_{\dot\alpha}(x) \end{pmatrix}\,. \tag{2.21}$$

Note that numerically the above matrix is equivalent to γ_0. From the group theoretical point of view, the above expressions suggest that Dirac spinors transform under the $(\frac{1}{2},0)\oplus(0,\frac{1}{2})$

[3] In order to show the correspondence one must explicitly include spinor indices.

representation of the Lorenrz group. Using eq. (2.19) the chiral projection operators in eq. (2.2) can be expressed as [1]:

$$P_L = \begin{pmatrix} \delta_\alpha{}^\beta & 0 \\ 0 & 0 \end{pmatrix}, \qquad P_R = \begin{pmatrix} 0 & 0 \\ 0 & \delta^{\dot\alpha}{}_{\dot\beta} \end{pmatrix}. \tag{2.22}$$

Eq. (2.22) then allows us to introduce the familiar four-component left and right-handed spinors, $\Psi_L(x)$ and $\Psi_R(x)$, defined as:

$$\Psi_L(x) \equiv P_L\Psi(x) = \begin{pmatrix} \chi_\alpha(x) \\ 0 \end{pmatrix}, \qquad \Psi_R(x) \equiv P_R\Psi(x) = \begin{pmatrix} 0 \\ \eta^{\dagger\dot\alpha}(x) \end{pmatrix}, \tag{2.23}$$

justifying their names as evident from eq. (2.23).

Using equations (2.19)-(2.23), one can now express frequently encountered Dirac spinor bilinears in the two component notation. Some of them are listed below:

$$\overline{\Psi}_i P_L \Psi_j = \eta_i\, \chi_j, \qquad \overline{\Psi}_i P_R \Psi_j = \chi_i^\dagger\, \eta_j^\dagger, \tag{2.24}$$

$$\overline{\Psi}_i \gamma^\mu P_L \Psi_j = \chi_i^\dagger \overline{\sigma}^\mu \chi_j, \qquad \overline{\Psi}_i \gamma^\mu P_R \Psi_j = \eta_i \sigma^\mu \eta_j^\dagger, \tag{2.25}$$

$$\overline{\Psi}_i \gamma^\mu \gamma_5 P_L \Psi_j = -\chi_i^\dagger \overline{\sigma}^\mu \chi_j, \qquad \overline{\Psi}_i \gamma^\mu \gamma_5 P_R \Psi_j = \eta_i \sigma^\mu \eta_j^\dagger, \tag{2.26}$$

with the spinor indices suppressed in accordance with eq. (2.11), and i,j run through the flavour or gauge degrees of freedom. These equations can be viewed as a recipe for quickly switching between the two- and four-component languages.

An in depth discussion of the correspondence between the two-component and four-component spinor notations can be found in Appendix G of Ref. [1].

2.3 Feynman rules and calculations in two-component spinor notation

In this subsection I briefly provide the basic Feynman rules, conventions and recipes for calculating matrix elements in two-component notation taken from Ref. [1].

External fermion and boson rules

Feynman rules for the two-component external state spinors (suppressing the momentum and spin arguments) are the following [1]:

- For an initial state (incoming) left-handed $(\frac{1}{2}, 0)$ fermion: x
- For an initial state (incoming) right-handed $(0, \frac{1}{2})$ fermion: $y^\dagger$
- For a final state (outgoing) left-handed $(\frac{1}{2}, 0)$ fermion: $x^\dagger$
- For a final state (outgoing) right-handed $(0, \frac{1}{2})$ fermion: y

x_α and y_α are *commuting* two-component spinor wave functions that satisfy the momentum-space Dirac equations [1]:

$$(p\cdot\overline{\sigma})^{\dot{\alpha}\beta}x_\beta = my^{\dagger\dot{\alpha}}\ , \qquad (p\cdot\sigma)_{\alpha\dot{\beta}}y^{\dagger\dot{\beta}} = mx_\alpha\ , \tag{2.27}$$

$$(p\cdot\sigma)_{\alpha\dot{\beta}}x^{\dagger\dot{\beta}} = -my_\alpha\ , \qquad (p\cdot\overline{\sigma})^{\dot{\alpha}\beta}y_\beta = -mx^{\dagger\dot{\alpha}}\ . \tag{2.28}$$

The mnemonic diagram [1] in Fig. 1 below summarizes these rules.

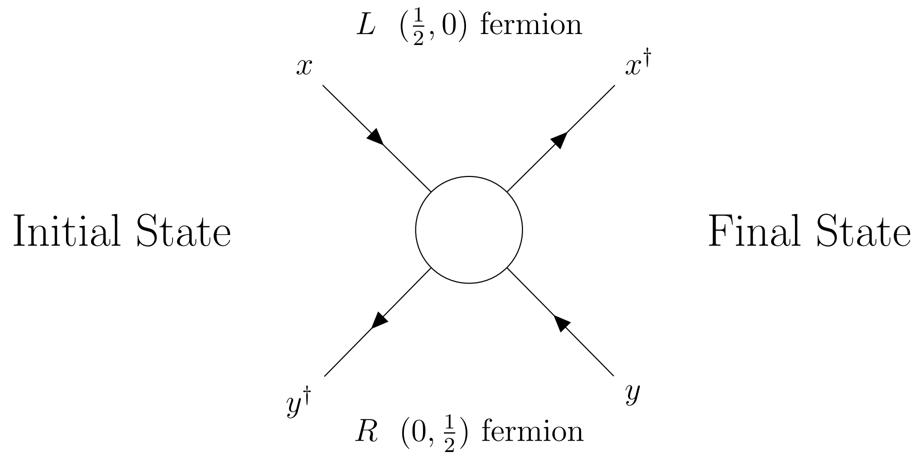


Figure 1: The external wave function spinors should be assigned as indicated here, for initial state and final state left-handed $(\frac{1}{2},0)$ and right-handed $(0,\frac{1}{2})$ fermions.

Note that the direction of the arrows on the external lines do *not* coincide with the flow of charge or fermion number, but instead correspond to their spinor index structure, with fields of undotted indices flowing into any vertex and fields of dotted indices flowing out of any vertex. The two-component Feynman rules for external bosons do not differ from the four-component counterparts. They are listed below for completeness' sake [1]:

- For an initial state (incoming) or final state (outgoing) spin-0 boson : 1
- For an initial state (incoming) spin-1 boson of momentum $\vec{\boldsymbol{p}}$ and helicity λ : $\varepsilon^\mu(\vec{\boldsymbol{p}},\,\lambda)$
- For a final state (outgoing) spin-1 boson of momentum $\vec{\boldsymbol{p}}$ and helicity λ : $\varepsilon^\mu(\vec{\boldsymbol{p}},\,\lambda)^*$

The treatment of propagators in two-component notation can be found in Ref. [1].

General structure and rules for Feynman diagrams

Here the basic recipe for matrix element calculations in two-component notation is presented. For the external lines we have [1]:

- If one starts a fermion line at an x or y external state spinor, it should have a raised undotted index. If one starts with an $x^\dagger$ or $y^\dagger$, it should have a lowered dotted spinor index. If one ends with an x or y external state spinor, it will have a lowered undotted index, while if one ends with an $x^\dagger$ or $y^\dagger$ spinor, it will have a raised dotted index. (2.29)

The following determines whether the σ or $\overline{\sigma}$ version of the rule for arrow-preserving fermion propagators and gauge vertices are employed [1]:

- For any scattering matrix amplitude, factors of σ and $\overline{\sigma}$ must alternate. If one or more factors of σ and/or $\overline{\sigma}$ are present, then x and y must be followed [preceded] by a σ [$\overline{\sigma}$], and $x^\dagger$ and $y^\dagger$ must be followed [preceded] by a $\overline{\sigma}$ [σ]. (2.30)
- Arrow-preserving propagator lines must be traversed in a direction parallel [antiparallel] to the arrowed line segment for the $\overline{\sigma}$ [σ] version of the propagator rule. (2.31)

Fermi-Dirac statistics yield the following rules [1]:

- Each closed fermion loop gets a factor of -1. (2.32)
- A relative minus sign is imposed between terms contributing to a given amplitude whenever the ordering of external state spinors (written left-to-right in a formula) differs by an odd permutation. (2.33)

The only thing that remains is to establish conventions for labelling Feynman diagrams in the two-component notation. Since Dirac fermions are made up of two distinct two-component spinor fields, there is an option of labelling fermion lines in Feynman rules and diagrams by particle names or by field names. In what follows, I will stick to conventions specified in Ref. [1] and label fermion lines with two-component fields rather than particle names. The exact labelling conventions for internal and external lines in Feynman diagrams and also in Feynman rules can be found in section 5 of Ref. [1].

3 The Standard Model in two-component notation

We can now use conventions and notations of the previous section to specify the fermionic part of the SM in two-component notation[4]. The fermionic content of the SM, listed in Table 1, is made up of three generations of quarks and leptons described by the two-component fermion fields. In Table 1 Y is the weak hypercharge, while T_3 is the third component of the weak isospin. Together they yield the electric charge via the relation: $Q = Y + T_3$ [1].

One can make connection to four-component Dirac notation using eq. (2.20). Associating $\chi \longleftrightarrow f$ and $\eta \longleftrightarrow \overline{f}$, a generic four-component Dirac fermion field is built in the following way [1]:[5]

$$f_{\rm D} \equiv \begin{pmatrix} f \\ \overline{f}^{\dagger} \end{pmatrix} . \tag{3.1}$$

The left- and right-handed projections are then

$$f_{{\rm D}L} \equiv \begin{pmatrix} f \\ 0 \end{pmatrix} , \qquad f_{{\rm D}R} \equiv \begin{pmatrix} 0 \\ \overline{f}^{\dagger} \end{pmatrix} . \tag{3.2}$$

The QCD interaction Lagrangian of the quarks with gluons is the following [1]:

$$\mathcal{L}_{\rm int} = -g_s A^{\mu}_{a} q^{\dagger mi}\, \overline{\sigma}_{\mu} (\boldsymbol{T}^a)_m{}^n q_{ni} + g_s A^{\mu}_{a} \overline{q}^{\dagger}_{ni}\, \overline{\sigma}_{\mu} (\boldsymbol{T}^a)_m{}^n \overline{q}^{mi} , \tag{3.3}$$

where q is a (mass eigenstate) quark field, m and n are $SU(3)$ color triplet indices, A^{μ}_{a} is the gluon field and T^a are the color generators in the triplet representation of $SU(3)$. The corresponding Feynman rules are given in Fig. 2 [1].

The electroweak interaction Lagrangian is given by [1]:

$$\begin{aligned} \mathcal{L}_{\rm int} = &- \frac{g}{\sqrt{2}} \left[\boldsymbol{K}_i{}^j u^{\dagger i} \overline{\sigma}^{\mu} d_j W^+_{\mu} + (\boldsymbol{K}^{\dagger})_i{}^j d^{\dagger i} \overline{\sigma}^{\mu} u_j W^-_{\mu} + \nu^{\dagger i} \overline{\sigma}^{\mu} \ell_i W^+_{\mu} + \ell^{\dagger i} \overline{\sigma}^{\mu} \nu_i W^-_{\mu} \right] \\ &- \frac{g}{c_W} \sum_{f=u,d,\nu,\ell} \left\{ (T^f_3 - s^2_W Q_f) f^{\dagger i} \overline{\sigma}^{\mu} f_i + s^2_W Q_f \overline{f}^{\dagger i} \overline{\sigma}^{\mu} \overline{f}_i \right\} Z_{\mu} \\ &- e \sum_{f=u,d,\ell} Q^f (f^{\dagger i} \overline{\sigma}^{\mu} f_i - \overline{f}^{\dagger i} \overline{\sigma}^{\mu} \overline{f}_i) A_{\mu} , \end{aligned} \tag{3.4}$$

where $s_W \equiv \sin\theta_W$, $c_W \equiv \cos\theta_W$, and i, j are generation indices. $\boldsymbol{K}$ is the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix. All the fermion fields above are the mass eigenstate fields. Fig. 3 [1] summarizes the Feynman rules .

[4] More detailed description of the SM (gauge and Higgs bosons, Higgs mechanism, mass diagonalizations, etc.) can be found in Appendix J of Ref. [1], Ref. [5] and the references in them.

[5] Usually four-component fermion fields are also denoted by f. For this reason I write the subscript "D" (short for Dirac) to differentiate the four-component field from the two-component $\left(\frac{1}{2}, 0\right)$ field f, even though it is fairly easy to differentiate between the two depending on the situation. One can also use capital letters to denote four-component Dirac fields when making connection to the two-component notation.

Two-component fields	$SU(3)_C, SU(2)_L, U(1)_Y$	T_3	$Q = T_3 + Y$
$Q_i \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}$	$(3, 2, \frac{1}{6})$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{2}{3}$ $-\frac{1}{3}$
$\overline{u}_i$	$(\overline{3}, 1, -\frac{2}{3})$	0	$-\frac{2}{3}$
$\overline{d}_i$	$(\overline{3}, 1, \frac{1}{3})$	0	$\frac{1}{3}$
$L_i \equiv \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix}$	$(1, 2, -\frac{1}{2})$	$\frac{1}{2}$ $-\frac{1}{2}$	0 -1
$\overline{\ell}_i$	$(1, 1, 1)$	0	1

Table 1: Fermions of the Standard Model with their $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers. $i = 1, 2, 3$ is a generation index. The bars on the two-component antifermion fields are part of their names, and do not denote some form of complex conjugation.

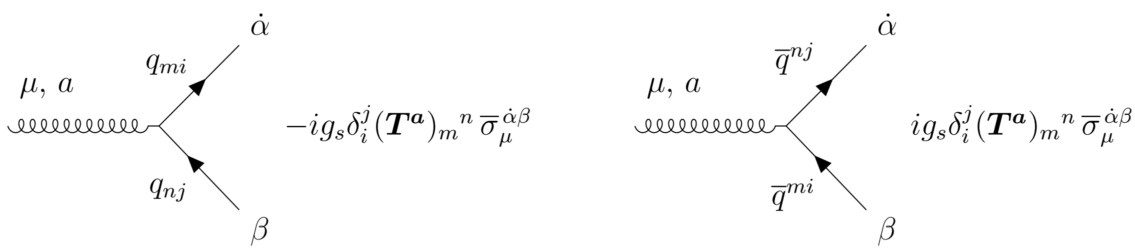


Figure 2: QCD Feynman vertex interaction rules involving the gluon and quark $q = u, d, c, s, t, b$. For each rule, a corresponding one with lowered spinor indices is obtained by $\overline{\sigma}_\mu^{\dot{\alpha}\beta} \to -\sigma_{\mu\beta\dot{\alpha}}$.

4 Supersymmetry and the Minimal Supersymmetric Standard Model

Supersymmetry (SUSY) is a spacetime symmetry that transforms a bosonic state into a fermionic state, and vice versa. Its algebraic structure involves a spin-$\frac{1}{2}$ anti-commuting spinor generator Q generating the transformations [2]

$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle\,, \qquad Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle\,. \tag{4.1}$$

Spinors being generally complex objects, the hermitian conjugate of Q is also a symmetry generator. Together they satisfy the following (anti-)commutation relations [2]:

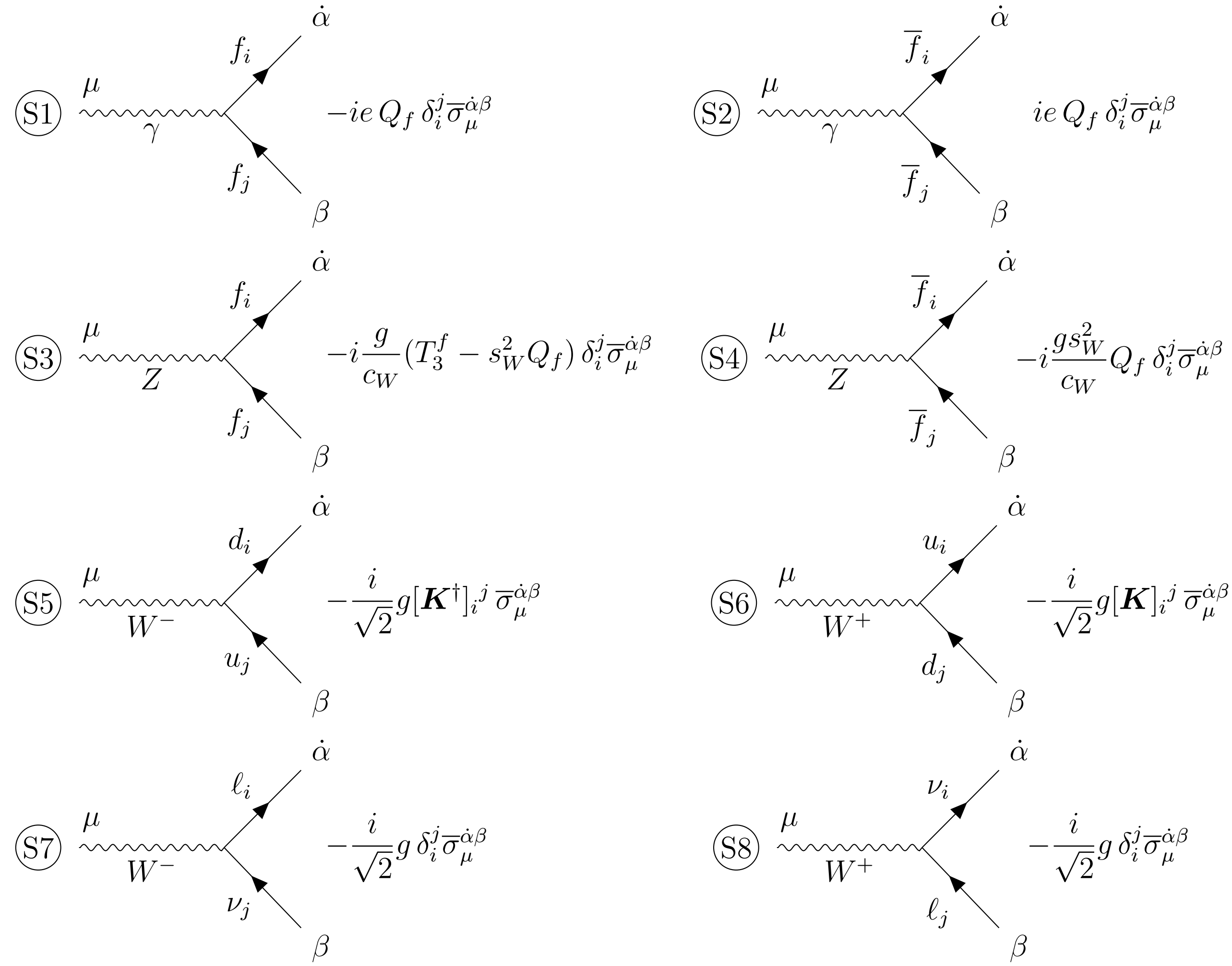


Figure 3: Feynman rules for the charged and neutral current interaction vertices. For the $W^\pm$ bosons, the charge indicated is flowing into the vertex. Q_f denotes the electric charge and $T_3^f = 1/2$ for $f = u, \nu$, while $T_3^f = -1/2$ for $f = d, \ell$. For each rule, a corresponding one with lowered spinor indices is obtained by $\overline{\sigma}_\mu^{\dot\alpha\beta} \to -\sigma_{\mu\beta\dot\alpha}$.

$$\{Q, Q^\dagger\} = -2\sigma_\mu P^\mu, \tag{4.2}$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \tag{4.3}$$
$$[Q, P^\mu] = [Q^\dagger, P^\mu] = 0, \tag{4.4}$$

where P^μ is the generator of space-time translations and the spinor indices on generators have been suppressed. The squared-mass operator P^2 also commutes with Q and $Q^\dagger$ [2]:

$$[Q, P^2] = [Q^\dagger, P^2] = 0, \tag{4.5}$$
$$[P^2, P^\mu] = 0. \tag{4.6}$$

The building blocks of the supersymmetric algebra are its irreducible representations called supermultiplets in which the single-particle states reside. Because of the nature of supersymmetry, each multipltet contains both fermionic and bosonic states, usually referred as superpartners of each other. Additionally, fermion and boson degrees of freedom are equal in any supermultiplet[6]. As a result of the equations (4.5) and (4.6) particles living in the same supermultiplet must have equal eigenvalues of P^2, that is equal masses. One can also show that the generators $Q, Q^\dagger$ commute with the generators of gauge transformations. Consequently, particles in the same supermultiplet have the same quantum numbers [2].

The simplest possible supermultiplets are the so-called chiral and gauge supermultiplets. A chiral supermultiplet consists of a single two-component Weyl fermion and 2 real scalars, conveniently integrated into a complex scalar field. A gauge supermultiplet contains a massless spin-1 gauge boson together with a massless spin-1/2 Weyl fermion. These massless fermions are called gauginos that, similar to their bosonic partners, transform in the adjoint representation of the gauge group, and thus have the same gauge transformation properties for the left- and right-handed components [2]. The SM fermions live in chiral supermultiplets in any supersymmetric extension of the Standard Model. Note that there are other types of supermultiplets, but they can always be reduced to combinations of chiral and gauge supermultiplets [2].

From this point I will focus on the the Minimal Sypersimmetric Standard Model (MSSM). First the particle content of the MSSM should be specified. As the SM quarks and leptons fit into chiral supermultiplets their partners are spin-0 bosons. These scalar bosons are conventionally called sfermions. The symbols used for the SM fermions also denote sfermions, but with a tilde ($\tilde{}$) on top. Since the left- and right-handed parts of the SM fermions are distinct two-component Weyl fermions, they live in separate supermultiplets, and thus have their own complex scalar partners.

Higgs scalar boson also falls into a chiral supermultiplet. Actually, In comparison to the SM, only one Higgs doublet is not enough in the MSSM. One needs two separate Higgs chiral supermultiplets with weak hypercharges $Y = 1/2$ and $Y = -1/2$ [2]. As will become evident from the MSSM superpotential later on, we need both to give masses to the SM fermions. These $S(U)_L$ doublet Higgs chiral supermultiplets are denoted by H_u and H_d, with hypercharges $Y = 1/2$ and $Y = -1/2$ respectively [2]. The fermionic partners of the Higgs scalars are called higgsinos. Their respective $S(U)_L$ doublet left-handed Weyl spinor fields are denoted by $\widetilde{H}_u$ and $\widetilde{H}_d$.

This is the chiral supermultiplet content of the MSSM. It is summarized in Table 2 [2].

[6]The proof can be found in Ref. [2].

Supermultiplets	spin 0 fields	spin 1/2 fields	$SU(3)_C, SU(2)_L, U(1)_Y$
Q_i	$(\widetilde{u}_L \quad \widetilde{d}_L)_i$	$(u \quad d)_i$	$(3,\, 2,\, \frac{1}{6})$
$\overline{u}_i$	$\widetilde{u}^\star_{Ri}$	$\overline{u}_i$	$(\overline{3},\, 1,\, -\frac{2}{3})$
$\overline{d}_i$	$\widetilde{d}^\star_{Ri}$	$\overline{d}_i$	$(\overline{3},\, 1,\, \frac{1}{3})$
L_i	$(\widetilde{\nu} \quad \widetilde{\ell}_L)_i$	$(\nu \quad \ell)_i$	$(1,\, 2,\, -\frac{1}{2})$
$\bar{e}_i$	$\widetilde{\ell}^\star_{Ri}$	$\overline{\ell}_i$	$(1,\, 1,\, 1)$
H_u	$(H_u^+ \quad H_u^0)$	$(\widetilde{H}_u^+ \quad \widetilde{H}_u^0)$	$(1,\, 2,\, +\frac{1}{2})$
H_d	$(H_d^0 \quad H_d^-)$	$(\widetilde{H}_d^0 \quad \widetilde{H}_d^-)$	$(1,\, 2,\, -\frac{1}{2})$

Table 2: Chiral supermultiplets in the Minimal Supersymmetric Standard Model. The spin-0 fields are complex scalars, and the spin-1/2 fields are left-handed two-component Weyl fermions. i=1,2,3 is a generation index. The bars on the fields are part of their names and do not denote any kind of conjugation.

The first column specifies the names of supermultiplets[7], the second and third columns list scalar and fermion fields respectively, and in the last column their transformation properties under the SM gauge group are given. Table 2 follows a standard convention according to which all chiral supermultiplets are defined in terms of left-handed Weyl spinors, so that the conjugates of the right-handed quarks and leptons (and their superpartners as well) appear in it [2].

The Standard Model gauge vector bosons reside in gauge supermultiplets together with their superpartner gauginos. The particle content of the gauge supermultiplets is briefly summarized in Table 3 [2]. Supersymmetric partners of gluons are called *gluinos* and are denoted by $\widetilde{g}$. The electroweak gauge bosons $W^\pm$, W^0 and B^0 are associated with Spin-1/2 superpartners $\widetilde{W}^\pm$, $\widetilde{W}^0$ and $\widetilde{B}^0$, called *winos* and *bino*. Note that after electroweak symmetry

[7]The symbols in the first column of Table 2 represent the whole supermultiplet. For example, L stands for the $SU(2)_L$-doublet chiral supermultiplet containing $\widetilde{\nu}$, ν (with $T_3 = 1/2$), and $\widetilde{\ell}_L$, ℓ_L (with $T_3 = -1/2$). In comparison, $\overline{e}$ stands for $SU(2)_L$-singlet chiral supermultiplet containing $\widetilde{\ell}^\star_R$, $\overline{\ell}$. Note that the same symbols can denote $SU(2)_L$-doublet fields. For instance, H_u, apart from representing the whole up-type Higgs supermultiplet, can also denote $SU(2)_L$-doublet scalar field $(H_u^+ \quad H_u^0)$, while L can stand for $SU(2)_L$-doublet spinor filed $(\nu \quad \ell)$ just like in Table 1. Usually it is straightforward to discern what the symbol denotes from the context. The same symbols can also designate superfields. It makes sense because they also contain as components all of the bosonic and fermionic fields within the corresponding supermultiplets. Nevertheless, I will always point out that we are dealing with the superfields when required.

Names	spin 1/2 fields	spin 1 fields	$SU(3)_C, SU(2)_L, U(1)_Y$
gluino, gluon	$\widetilde{g}$	g	(8, 1, 0)
winos, W bosons	$\widetilde{W}^{\pm} \quad \widetilde{W}^0$	$W^{\pm} \quad W^0$	(1, 3, 0)
bino, B boson	$\widetilde{B}^0$	B^0	(1, 1, 0)

Table 3: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

breaking, just as W^0 and B^0 interaction eigenstates mix to give mass eigenstate γ and Z^0, $\widetilde{W}^0$ and $\widetilde{B}^0$ will also mix giving the so-called *photino* $\widetilde{\gamma}$ and *zino* $\widetilde{Z^0}$.

To sum up, Tables 2 and 3 make up all of the particle content of the MSSM. Next we have to specify the superpotential as it governs the supersymmetric interactions bewteen the chiral superpartners.

The superpotential of the MSSM is [2]

$$W_{\text{MSSM}} = \overline{u}\mathbf{y}_u QH_u - \overline{d}\mathbf{y}_d QH_d - \overline{e}\mathbf{y}_e LH_d + \mu H_u H_d, \tag{4.7}$$

where Q, L, H_u, H_d, $\overline{e}$, $\overline{u}$, $\overline{d}$ are chiral superfields corresponding to the chiral supermultiplets given in Table 2 and all the gauge and family indices have been suppressed. A superfield is a single object that contains as components all of the bosonic and fermionic fields within the corresponding supermultiplet. The gauge quantum numbers and the mass dimension of a chiral superfield are the same as that of its scalar component [8][2]. $\mathbf{y}_e$, $\mathbf{y}_u$, $\mathbf{y}_d$, 3×3 matrices in the family space, are the dimensionless Yukawa coupling parameters.

Note that in the superpotential only terms holomorphic in the chiral superfields treated as complex variables (no complex conjugates should appear) are admitted. This implies that terms like $H_u^{\star}H_u$ can not be part of the superpotential [2]. This constraint also makes it evident why one needs two Higgs supermultiplets to give masses to SM fermions. Since the superpotential is holomorphic, only H_u can give masses to the up-type quarks and only H_d can generate down-type quark and charged lepton masses after electroweak symmetry breaking. The different interactions and vertices produced by superpotential (4.7) and, additionally, the treatment of the gauge interactions as well can be found in Ref. [2].

Tables 2 and 3 and superpotential (4.7) are the basic building blocks of the MSSM. They contain the particle spectrum and all the possible supersymmetry conserving terms that are

[8]The detailed discussion of superspace, superfields and the treatment of supersymmetry in this formalism can be found in section 4 of Ref. [2]

compatible with gauge invariance and R-parity conservation in the MSSM. So new question arises: what is R-parity, and why do we need it at all in the MSSM?

5 R-parity

The concept of R-parity is closely related to the conservation of baryon number B and lepton number L. One of the problems that can arise with the introduction of sypersymmetry concerns the definition of B and L [3] that are carried by Dirac fermions. These numbers appear to be fermionic in nature, since they are carried only by fundamental fermions as far as we are concerned. This can not be the case any more in any supersymmetric extension of the SM, because the fundamental fermions of the SM live in chiral supermultiplets together with their superpartners, spin-0 bosons, which in supersymmetry have the fundamental status themselves. Since the superpartners must share the same quantum numbers, we then have no other choice but to attribute baryon and lepton numbers to fundamental bosons as well. In the MSSM these boson are squarks and sleptons. But now B and L do not have to be necessarily conserved due to possibly new interactions of supersymmetric theories [3]. Indeed the new bosons, if allowed to be exchanged between ordinary SM particles conjointly with the SM gauge bosons, would change the entire picture, allowing extra contributions to the normal processes and generating unwanted interactions mediated by scalar bosons. In contrast, SM interactions are due to the exchange of spin-1 gauge bosons. To remedy these problems a continuous R-symmetry $U(1)_R$ acting on the susy generator was introduced [3]. It allows for an additive conserved quantum number, R, and different values of it are carried by superpartners inside the multiplets of supersymmetry. R-parity is the discrete version (Z_2 subgroup) of $U(1)_R$, is connected to B and L as will become evident below, and successfully forbids the unwanted direct exchanges of squarks and sleptons between SM fermions when one imposes its conservation.

R-symmetry is a global $U(1)_R$ symmetry under which some supersymmetric Lagrangians are invariant. The defining feature of a continuous R-symmetry is that the anti-commuting coordinates θ and $\theta^\dagger$ are oppositely charged under it [2]:

$$\theta \to e^{-i\alpha}\theta, \qquad \theta^\dagger \to e^{i\alpha}\theta^\dagger, \tag{5.1}$$

where α parametrizes the $U(1)$ transformation. Eq. (5.1)[9] leads to [2]:

$$Q \to e^{i\alpha}Q, \qquad Q^\dagger \to e^{-i\alpha}Q, \tag{5.2}$$

[9]Explicit expressions of susy generators in terms of superspace coordinates can be found in section 4 of Ref. [2].

which in turn implies that susy generators do not commute with $U(1)_R$ generator R [2]:

$$[R,Q] = Q, \qquad [R,Q^\dagger] = -Q^\dagger. \tag{5.3}$$

Consequently, R-symmetry distinguishes between bosons and fermions belonging to the same supermultuplets as they always carry different values of R charges.

In a theory invariant under an R-symmetry a general transformation rule of any superfield $S(x,\, \theta,\, \theta^\dagger)$ of charge R is given by [2]:

$$S(x,\, \theta,\, \theta^\dagger) \to e^{iR\alpha} S(x,\, e^{-i\alpha}\theta,\, e^{i\alpha}\theta^\dagger). \tag{5.4}$$

In a component form we have [2]:

$$\phi \to e^{iR_\phi \alpha}\phi, \qquad \psi \to e^{i(R_\phi - 1)\alpha}\psi, \tag{5.5}$$

where ϕ and ψ are the scalar and fermion fields of some chiral supermultiplet and R_ϕ is a charge assigned to this chiral supermultiplet. The components of a related anti-chiral supermultiplet carry the opposite charges. Gauge superfields being real are not charged under $U(1)_R$ at all. For their components it follows in Wess-Zumino gauge that

$$A_\mu \to A_\mu, \qquad \lambda \to e^{i\alpha}\lambda, \tag{5.6}$$

where A_μ and λ are gauge boson and gaugino fields respectively [2].

We can define R-transformation to not act on ordinary particles meaning that they have $R = 0$ [3]. Gauge bosons already satisfy this condition as evident in eq. (5.6). We can extend this definition to chiral Higgs , quark and lepton superfields, so that Higgs scalars and SM fermions all have $R = 0$. This way all ordinary particles get $R = 0$, while their superpartners, namely gauginos, higgsinos and sfermions will have $R = \pm 1$ (for example as eq. (5.5) suggests $\widetilde{\ell}_L$, $\widetilde{q}_L$ have $R = +1$, while $\widetilde{\ell}_R$, $\widetilde{q}_R$ have $R = -1$). These assignments allow us to define R-parity as the parity of the additive quantum number R associated with $U(1)_R$ [3]:

$$\boxed{R_p \equiv (-1)^R = \begin{cases} +1\,, & \text{for ordinary particles,} \\ -1\,, & \text{for their superpartners.} \end{cases}} \tag{5.7}$$

Thus we end up with two separate sectors of R-even and R-odd particles, with R-even ($R_p = +1$) particles including SM fermions, gauge bosons and the Higgs bosons of H_u and H_d, and R-odd ($R_p = -1$) particles encompassing their superpartners. The definition (5.7) corresponds to restricting the $U(1)_R$ symmetry to its Z_2 discrete subgroup by constraining parameter α to integer multiples of π.

In order to connect R-parity to B and L we have to first discuss the so-called matter parity. It is a multiplicatively conserved quantum number defined as [2]

$$P_M \equiv (-1)^{3B+L}, \tag{5.8}$$

which was initially used to constrain superpotential to be an even function of the quark and lepton superfields, thus to be able to recover B and L conservation laws, and avoid direct Yukawa exchanges of spin-0 bosons between ordinary SM fermions. Using the above definition of P_M, one can now connect R-parity to B and L by re-expressing it in the following way [3]:

$$\boxed{R_p = (-1)^{2s}(-1)^{3B+L},} \tag{5.9}$$

s being the spin of the particle. Note that $(-1)^{2s}$ coincides with $(-1)^{3B+L}$ for all ordinary particles, hence giving $R_p = +1$ when multiplied. Because the spin of the superpartners differ by 1/2, $(-1)^{2s}$ and therefore their R_p charges are exactly opposite. This means that definitions (5.9) and (5.7) are equivalent.

To see how R_p conservation forbids possible additional unwanted interaction terms, let's first find out if there are such terms at all in the MSSM superpotential. Indeed, as it turns out there are other terms both gauge-invariant and holomorphic in the chiral superfields, but are not part of the MSSM because they violate either B or L [2]. These additional terms are given below in two extra superpotentils [2]:

$$W_{\Delta L=1} = \tfrac{1}{2}\lambda_{ijk}L_iL_j\overline{e}_k + \lambda'_{ijk}L_iQ_j\overline{d}_k + \mu'_iL_iH_u, \tag{5.10}$$

$$W_{\Delta B=1} = \tfrac{1}{2}\lambda''_{ijk}\overline{u}_i\overline{d}_j\overline{d}_k, \tag{5.11}$$

where $i, j, k = 1, 2, 3$ are family indices . L_i have $L = +1$, $\overline{e}_i$ have $L = -1$, and $L = 0$ for the other supermultiplets. Q_i carry $B = +\frac{1}{3}$, $\overline{u}_i$ and d_i are assigned $B = -\frac{1}{3}$, while for the rest $B = 0$. It is trivial to check that the terms in equations (5.10) and (5.11) indeed violate L and B respectively by 1 unit. Gauge indices have been suppressed, which means that one has for example $L_iL_j \equiv \epsilon^{\alpha\beta}L_{i\alpha}L_{j\beta}$ and $\epsilon^{abc}\,\overline{u}_{ia}\overline{d}_{jb}\overline{d}_{kc}$, where $\alpha, \beta = 1, 2$ are $SU(2)_L$ indices and $a, b, c = 1, 2, 3$ are $SU(3)_C$ indices [3]. Because of the antisymmetry of $\epsilon^{\alpha\beta}$ and ϵ^{abc}, λ_{ijk} are antisymmetric with respect to their first two indices, while λ''_{ijk} is with respect to their last two indices:

$$\lambda_{ijk} = -\lambda_{jik}, \qquad \lambda''_{ijk} = -\lambda''_{ikj}. \tag{5.12}$$

A candidate term in the Lagrangian (or in the superpotential) is allowed only if the product of R_p (or P_M) for the fields in it is +1 [2]. The reason for sticking to R_p instead of matter-parity is phenomenological, since all the SM particles and the Higgs bosons carry $R_p = +1$, while their superpartners, with none of them yet discovered, have $R_p = -1$. With the R_p values already assigned it is easy to check that the terms in equations (5.10) and

(5.11) are indeed forbidden if R_p is exactly conserved. But if R-parity is violated, then the MSSM is enhanced with $W_{\Delta L/B=1}$ superpotentials. They generate a variety of terms. I will focus only on the trilinear, Yukawa-like interactions involving two fermions and one scalar boson. There are also bilinear terms involving fermions coming from μ'_i terms in $W_{\Delta L=1}$, scalar interactions both R_p conserving and violating, as well as $\not{R}_p$ soft terms[10].

In order to obtain Yukawa interactions we have to look for the F-terms [2] in the $\not{R}_p$ superpotentials. The following identity involving the anti-commuting coordinates will be useful [2]:

$$(\theta\xi)(\theta\chi) = -\frac{1}{2}(\theta\theta)(\xi\chi). \tag{5.13}$$

Chiral superfields can be expanded in the following way [2]:

$$\Phi = \phi(y) + \sqrt{2}\,\theta\psi(y) + \theta\theta F(y),$$

where $\phi(y)$ is a scalar field, $\psi(y)$ is a fermion field, and $F(y)$ is an auxiliary complex scalar field. Let's now look at, for example, the λ terms in the $W_{\Delta L=1}$. Chiral superfields involved are $SU(2)_L$ lepton doublets L_i and $SU(2)_L$ lepton singlets $\overline{e}_i$:

$$L_i = \begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{\ell}_{Li} \end{pmatrix} + \sqrt{2}\,\theta \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix} + \theta\theta F_{Li}\,, \qquad \overline{e}_i = \widetilde{e}^{\star}_{Ri} + \sqrt{2}\,\theta\overline{e}_i + \theta\theta F_{ei}\,,$$

$L_i L_j \overline{e}_k$ is then:

$$L_i L_j \overline{e}_k = \left[\begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{\ell}_{Li} \end{pmatrix} + \sqrt{2}\,\theta \begin{pmatrix} \nu_i \\ \ell_i \end{pmatrix} + \theta\theta F_{Li}\right] \left[\begin{pmatrix} \widetilde{\nu}_j \\ \widetilde{\ell}_{Lj} \end{pmatrix} + \sqrt{2}\,\theta \begin{pmatrix} \nu_j \\ \ell_j \end{pmatrix} + \theta\theta F_{Lj}\right] \left[\widetilde{e}^{\star}_{Rk} + \sqrt{2}\,\theta\overline{e}_k + \theta\theta F_{ek}\right] \tag{5.14}$$

One of the F-terms can be obtained by combining the first term in the first bracket, the second term in the second bracket, and the second term in the third bracket:

$$\begin{aligned} 2\,\epsilon^{\alpha\beta} \begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{\ell}_{Li} \end{pmatrix}_\alpha \begin{pmatrix} \theta\nu_j \\ \theta\ell_j \end{pmatrix}_\beta (\theta\overline{\ell}_k) &= 2\,\epsilon^{12} \begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{\ell}_{Li} \end{pmatrix}_1 \begin{pmatrix} \theta\nu_j \\ \theta\ell_j \end{pmatrix}_2 (\theta\overline{\ell}_k) + 2\,\epsilon^{21} \begin{pmatrix} \widetilde{\nu}_i \\ \widetilde{\ell}_{Li} \end{pmatrix}_2 \begin{pmatrix} \theta\nu_j \\ \theta\ell_j \end{pmatrix}_1 (\theta\overline{\ell}_k) \\ &= 2\left[\widetilde{\nu}_i\,(\theta\ell_j)(\theta\overline{\ell}_k) - \widetilde{\ell}_{Li}\,(\theta\nu_j)(\theta\overline{\ell}_k)\right] \end{aligned}$$

Now we can use eq. (5.13) to get to the final form:

$$2\left[\widetilde{\nu}_i\,(\theta\ell_j)(\theta\overline{\ell}_k) - \widetilde{\ell}_{Li}\,(\theta\nu_j)(\theta\overline{\ell}_k)\right] = -(\theta\theta)(\ell_j\overline{\ell}_k)\widetilde{\nu}_i + (\theta\theta)(\nu_j\overline{\ell}_k)\widetilde{\ell}_{Li}.$$

So taking the F-term will yield the following interaction terms: $\widetilde{\nu}_i\ell_j\overline{\ell}_k$ and $\widetilde{\ell}_{Li}\nu_j\overline{\ell}_k$. There are also the same terms but with $i \leftrightarrow j$ coming from the product of the second term in the

[10]All these interaction terms can be found in section 2 of [3].

first bracket of eq. (5.14), the first term of the second bracket, and the second term of the third bracket. Additionally more F-terms can be obtained by combining the second terms in the first two brackets with the first term of the third bracket. The same steps can be done for the λ' and λ'' terms in the $\not{R}_p$ superpotentials. For each Lagrangian term there is also its complex conjugate. Altogether we get the following $\not{R}_p$ trilinear Lagrangian interaction terms [1]:

$$\mathcal{L}_{LL\bar{e}} = -\frac{1}{2}\lambda_{ijk}\left(\widetilde{\ell}^*_{Rk}\nu_i\ell_j + \widetilde{\nu}_i\ell_j\overline{\ell}_k + \widetilde{\ell}_{Lj}\overline{\ell}_k\nu_i - \widetilde{\ell}^*_{Rk}\ell_i\nu_j - \widetilde{\nu}_j\overline{\ell}_k\ell_i - \widetilde{\ell}_{Li}\nu_j\overline{\ell}_k\right) + \text{h.c.}\,, \tag{5.15}$$

$$\mathcal{L}_{LQ\bar{d}} = -\lambda'_{ijk}\left(\widetilde{d}^*_{Rk}\nu_i d_j + \widetilde{\nu}_i d_j\overline{d}_k + \widetilde{d}_{Lj}\overline{d}_k\nu_i - \widetilde{d}^*_{Rk}\ell_i u_j - \widetilde{u}_{Lj}\overline{d}_k\ell_i - \widetilde{\ell}_{Li}u_j\overline{d}_k\right) + \text{h.c.}\,, \tag{5.16}$$

$$\mathcal{L}_{\overline{u}\,\overline{d}\,\overline{d}} = -\frac{1}{2}\lambda''_{ijk}\epsilon_{pqr}\left[\widetilde{u}^{p*}_{Ri}\overline{d}^{q}_j\overline{d}^{r}_k + \widetilde{d}^{q*}_{Rj}\overline{u}^{p}_i\overline{d}^{r}_k + \widetilde{d}^{r*}_{Rk}\overline{u}^{p}_i\overline{d}^{q}_j\right] + \text{h.c.}\,, \tag{5.17}$$

with the Feynman vertex rules given in Fig.s 4, 5 and 6 [1].

These Lagrangians contain terms that yield extra contributions to the usual Standard Model processes, as well as generate new ones that are forbidden in the SM. By computing $\not{R}_P$ contributions and using experimental values for different observables, indirect bounds on trilinear λ couplings can be placed. This is precisely the goal of the next section.

Before going to the next topic let's briefly discuss the reason why we limit ourselves to the discrete R-parity and discard the continuous $U(1)_R$ symmetry. Actually, $U(1)_R$ happens to be a symmetry of all four necessary basic building blocks of the MSSM: the Lagrangian density for the gauge superfields responsible for strong and electroweak interactions, the gauge interactions of the quark and lepton superfields, the gauge interactions of the two chiral doublet Higgs superfields H_u and H_d responsible for the electroweak symmetry breaking, and the "super-Yukawa" interactions, coming from W_{MSSM}, responsible for quark and lepton masses [3]. So why do we abandon $U(1)_R$ in favour of R-parity? The reason is that an unbroken $U(1)_R$ constrains gauginos to remain massless, even after spontaneous breaking of supersymmetry. A Majorana gaugino mass term $\frac{1}{2}M_\lambda\lambda\lambda$ will always break the continuous $U(1)_R$ symmetry and, consequently, it will be absent in the susy breaking Lagrangian of the MSSM. Since we want supersymmetry to be necessarily broken only the discrete version of $U(1)_R$, namely R-parity, can be tolerated.

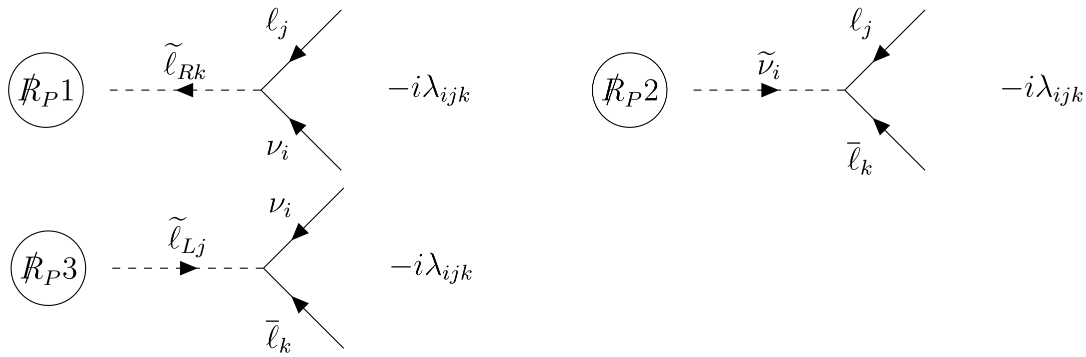


Figure 4: Feynman rules for the Yukawa couplings of two-component fermions due to the supersymmetric, R-parity-violating Yukawa Lagrangian $\mathcal{L}_{LL\overline{e}}$. For each diagram, there is another with all arrows reversed and $\lambda_{ijk} \to \lambda^*_{ijk}$.

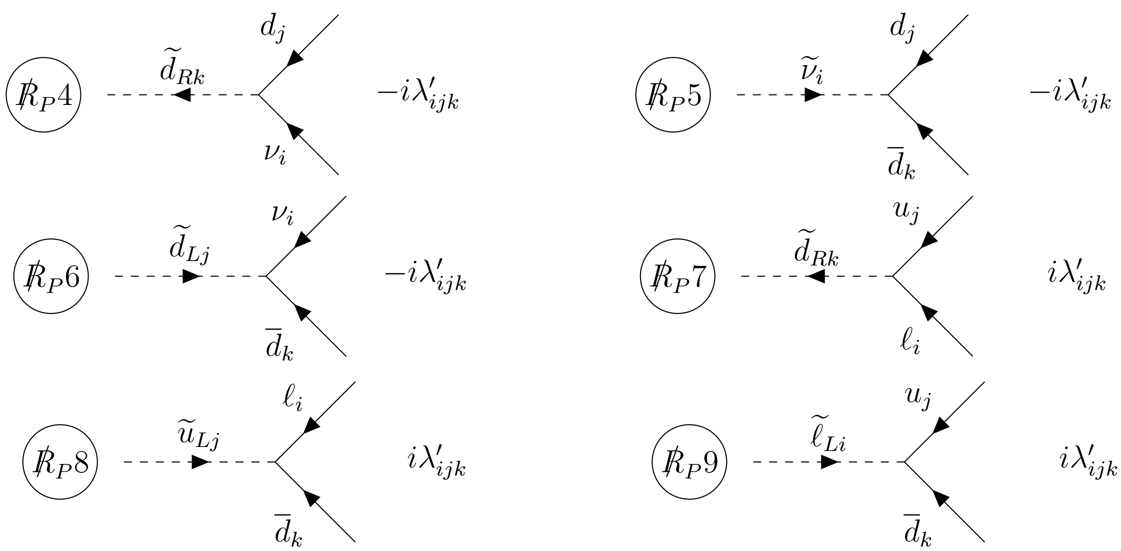


Figure 5: Feynman rules for the Yukawa couplings of two-component fermions for the supersymmetric, R-parity-violating Yukawa Lagrangian $\mathcal{L}_{LQ\overline{d}}$. For each diagram, there is another with all arrows reversed and $\lambda'_{ijk} \to \lambda'^*_{ijk}$.

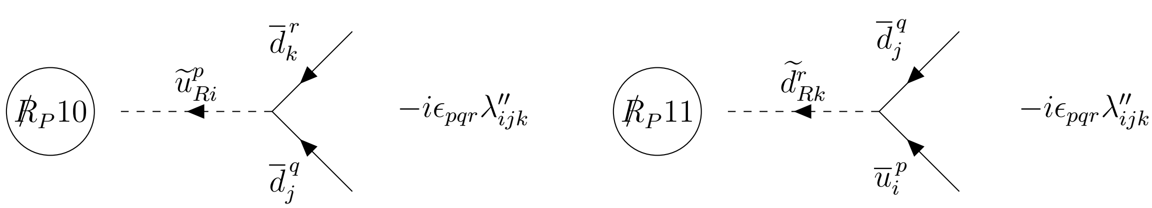


Figure 6: Feynman rules for the Yukawa couplings of two-component fermions due to the supersymmetric, R-parity-violating Yukawa Lagrangian $\mathcal{L}_{\overline{u}\overline{d}\overline{d}}$. For each diagram, there is another with all arrows reversed and $\lambda''_{ijk} \to \lambda''^*_{ijk}$.

6 Single-Coupling Bounds on the Trilinear λ Couplings

In this section I go through various low energy processes mainly in the charged current (CC) and neutral current (NC) sectors and obtain bounds on the trilinear λ couplings from various observables. These bounds are single-coupling bounds derived under the so-called single coupling dominance hypothesis, where a single $\not{R}_P$ coupling dominates over all the others, with typical orders of magnitude being $\lambda, \lambda', \lambda'' < (10^{-2} - 10^{-1}) \times \dfrac{\widetilde{m}}{100\text{GeV}}$ and involving a linear dependence on the exchanged sfermion masses [3].

To simplify notation, it is useful to define the following auxiliary parameters [4]:

$$r_{ijk}(\widetilde{\ell}) = \left(\frac{M_W^2}{g^2}\right)\frac{|\lambda_{ijk}|^2}{m_{\widetilde{\ell}}^2}, \qquad r'_{ijk}(\widetilde{q}) = \left(\frac{M_W^2}{g^2}\right)\frac{|\lambda'_{ijk}|^2}{m_{\widetilde{q}}^2}. \tag{6.1}$$

Shifts in observables in the CC and NC sectors will be expressed in terms of these dimensionless quantities.

6.1 Charged Current Universality In Lepton Sector

One of the distinctive properties of the standard electroweak theory is the universality of lepton and quark couplings to $W^{\pm}$ bosons. The $LL\overline{e}$ and $LQ\overline{d}$ operators lead to universality violations. Consequently, stringent bounds on $\not{R}_P$ couplings can be placed by precision measurements of the CC universality. For example, the λ_{12k} couplings induce additional contributions to the muon decay $\mu^- \to \nu_\mu e^- \overline{\nu}_e$, involving vertex ($\not{R}_P 1$) and its counterpart with all the arrow reversed as shown in Fig. 7(b).

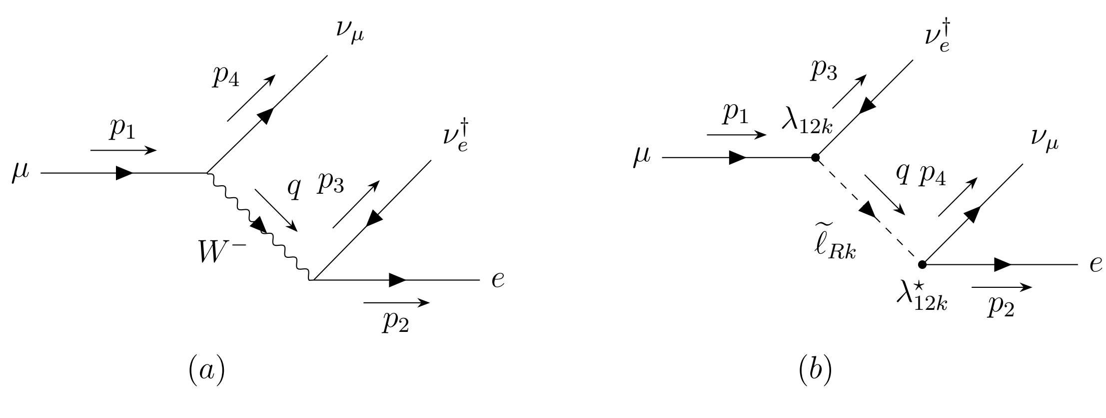


Figure 7: Contributions to the muon decay from (a) the Standard Model and (b) an $\not{R}_P$ operator. $q^2 = t = (p_1 - p_{4/3})^2$ in (a)/(b).

The effective tree-level Fermi coupling G_F will be modified by the $\not{R}_P$ operator. In order to get the $\not{R}_P$ contributions let's evaluate matrix elements at low energies for the above diagrams:

$$\begin{aligned}
i\mathcal{M}^{\not R_P} &= \left\{x_1^{\alpha}\left[-i\lambda_{12k}\,\delta_{\alpha}{}^{\beta}\right]y_{3\beta}\right\}\left(\frac{i}{t-m^2_{\widetilde{\ell}_{Rk}}}\right)\left\{x^{\dagger}_{2\dot\alpha}\left[i\lambda^*_{12k}\,\delta^{\dot\alpha}{}_{\dot\beta}\right]x_4^{\dagger\dot\beta}\right\} \\
&= \frac{i\,|\lambda_{12k}|^2}{(t-m^2_{\widetilde{\ell}_{Rk}})}\{x_1y_3\}\{x_2^{\dagger}x_4^{\dagger}\} \\
&\overset{t\ll m^2_{\widetilde{\ell}}}{\approx} -\frac{i\,|\lambda_{12k}|^2}{m^2_{\widetilde{\ell}_{Rk}}}\{x_1y_3\}\{x_2^{\dagger}x_4^{\dagger}\} \\
&\overset{(2.16)}{=} -\frac{i\,|\lambda_{12k}|^2}{2m^2_{\widetilde{\ell}_{Rk}}}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}_{\mu}y_3\} \\
\Longrightarrow\ \mathcal{M}^{\not R_P} &= -\frac{|\lambda_{12k}|^2}{2m^2_{\widetilde{\ell}_{Rk}}}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}_{\mu}y_3\}.
\end{aligned}\tag{6.2}$$

Using vertex rules (S7) and (S8), I get the following low energy SM matrix element:

$$\begin{aligned}
i\mathcal{M}^{SM} &= \left\{x_1^{\alpha}\left[\frac{ig\sigma_{\mu\alpha\dot\alpha}}{\sqrt{2}}\right]x_4^{\dagger\dot\alpha}\right\}\left(\frac{-ig^{\mu\nu}}{t-M_W^2}\right)\left\{x^{\dagger}_{2\dot\beta}\left[\frac{-ig\overline{\sigma}_{\nu}^{\dot\beta\beta}}{\sqrt{2}}\right]y_{3\beta}\right\} \\
&= \frac{-ig^2}{2\,(t-M_W^2)}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}^{\mu}y_3\} \\
&\overset{t\ll M_W^2}{\approx} \frac{ig^2}{2M_W^2}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}^{\mu}y_3\} \\
\Longrightarrow\ \mathcal{M}^{SM} &= \frac{g^2}{2M_W^2}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}^{\mu}y_3\}.
\end{aligned}\tag{6.3}$$

So we have[11]:

$$\begin{cases}
\mathcal{M}^{SM} = \dfrac{g^2}{2M_W^2}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}^{\mu}y_3\}. \\
- \\
\mathcal{M}^{\not R_P} = -\dfrac{|\lambda_{12k}|^2}{2m^2_{\widetilde{\ell}_{Rk}}}\{x_1\sigma^{\mu}x_4^{\dagger}\}\{x_2^{\dagger}\overline{\sigma}_{\mu}y_3\}.
\end{cases}$$

Thus $\mathcal{M}^{Total} = \mathcal{M}^{SM} - \mathcal{M}^{\not R_P}$, so that G_F becomes:

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} + \frac{|\lambda_{12k}|^2}{8m^2_{\widetilde{\ell}_{Rk}}} = \frac{g^2}{8M_W^2}\left[1+\left(\frac{M_W^2}{g^2}\right)\frac{|\lambda_{12k}|^2}{m^2_{\widetilde{\ell}_{Rk}}}\right] = \frac{g^2}{8M_W^2}\left[1+r_{12k}(\widetilde{\ell}_{Rk})\right]. \tag{6.4}$$

This shift will affect other observables through Fermi coupling G_F.

[11] relative minus sign due to the ordering of the external fermions. It does not really matter which matrix element gets a minus sign in front, only the relative sign matters.

Bounds on the $LL\bar{e}$ couplings can be obtained by comparing the measured ratios of the CC decay widths to their Standard Model expectations. One of these ratios is defined in the following way [4]:

$$R_{\tau\mu} \equiv \frac{\Gamma(\tau \to \mu\nu\overline{\nu})}{\Gamma(\mu \to e\nu\overline{\nu})}. \tag{6.5}$$

The diagram and the matrix element calculation for $\tau \to \mu\nu\overline{\nu}$ are similar to the muon decay counterparts after replacing $\mu \to \tau$, $e \to \mu$, $\nu_\mu \to \nu_\tau$, $\nu_e^\dagger \to \nu_\mu^\dagger$ and $\lambda_{12k} \to \lambda_{23k}$. Since $\Gamma \propto |\mathcal{M}|^2$ we get: $\Gamma(\tau \to \mu\nu\overline{\nu}) \propto (1 + r_{23k}(\widetilde{\ell}_{Rk}))^2 \approx 1 + 2\, r_{23k}$ and similarly $\Gamma(\mu \to e\nu\overline{\nu}) \propto (1 + r_{12k}(\widetilde{e}_{Rk}))^2 \approx 1 + 2\, r_{12k}$. The expression for $R_{\tau\mu}$ is then

$$\begin{aligned} R_{\tau\mu} &\approx R_{\tau\mu}^{SM}\left(\frac{1+2\,r_{23k}}{1+2\,r_{12k}}\right) = R_{\tau\mu}^{SM}\frac{(1+2\,r_{23k})(1-2\,r_{12k})}{(1+2\,r_{12k})(1-2\,r_{12k})} = R_{\tau\mu}^{SM}\frac{(1+2\,r_{23k})(1-2\,r_{12k})}{1-4r_{12k}^2} \\ &\approx R_{\tau\mu}^{SM}\left[1+2\,r_{23k}(\widetilde{\ell}_{Rk}) - 2\,r_{12k}(\widetilde{\ell}_{Rk}) + \mathcal{O}(r^2)\right], \end{aligned} \tag{6.6}$$

where in the last step I have neglected the $r_{12k}^2 \ll 1$ terms in the denominator.

The measured branching ratios are [6]

$$\Gamma_{\mu\to e} \approx \Gamma_\mu^{\text{Total}}, \qquad \frac{\Gamma_{\tau\to\mu}}{\Gamma_\tau^{\text{Total}}} = (17.39 \pm 0.04) \times 10^{-2}.$$

The lifetimes of μ and τ are $\tau_\mu \approx 2.197 \times 10^{-6}$ and $\tau_\tau \approx (290.3 \pm 0.5) \times 10^{-15}$ respectively, while $m_\mu \approx 105.6584$ MeV and $m_\tau = 1776.86 \pm 0.12$ MeV [6]. Using these numbers I get that

$$R_{\tau\mu} = (131.4183 \pm 0.3023) \times 10^4. \tag{6.7}$$

The SM expression for $R_{\tau\mu}$ including radiative corrections is [7]

$$R_{\tau\mu}^{SM} = \frac{m_\tau^5}{m_\mu^5}\frac{f(m_\mu^2/m_\tau^2)}{f(m_e^2/m_\mu^2)}\frac{\delta_W^\tau}{\delta_W^\mu}\frac{\delta_\gamma^\tau}{\delta_\gamma^\mu} = 1.309 \times 10^6, \tag{6.8}$$

where $f(x)$ is the phase space factor

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \tag{6.9}$$

δ_W^ℓ is the W-boson propagator correction

$$\delta_W^\ell = 1 + \frac{3}{5}\frac{m_\ell^2}{M_W^2}, \tag{6.10}$$

and δ_γ^ℓ is one-loop correction from photons

$$\delta_\gamma^\ell = 1 + \frac{\alpha(m_\ell)}{2\pi}\left(\frac{25}{4} - \pi^2\right). \tag{6.11}$$

Using the results of eq.s (6.7) and (6.8) I get that

$$\frac{R_{\tau\mu}}{R_{\tau\mu}^{SM}} = 1.0040 \pm 0.0023.$$

Plugging this number back in eq. (6.6) then yields the following single bounds:

$$|\lambda_{23k}| = 0.04 \pm 0.01\,(< 0.05)\left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma) \tag{6.12}$$

$$|\lambda_{12k}| < 0.01\left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right). \qquad 2\sigma \tag{6.13}$$

Eq. (6.6) excludes λ_{12k} at 1σ level.

Another measure of the $LL\bar{e}$ operators comes from the following ratio [4]:

$$R_\tau \equiv \frac{\Gamma(\tau \to e\overline{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mu\overline{\nu}_\mu\nu_\tau)}. \tag{6.14}$$

R_τ is modified by $\not{R}_P$ operators analogous to $R_{\tau\mu}$, with the replacement of r_{23k} and r_{12k} by r_{13k} and r_{23k}, respectively, giving:

$$R_\tau \approx R_\tau^{SM}\left[1 + 2\,r_{13k}(\widetilde{\ell}_{Rk}) - 2\,r_{23k}(\widetilde{\ell}_{Rk}) + \mathcal{O}(r^2)\right]. \tag{6.15}$$

The measured branching ratios are [6]

$$\frac{\Gamma_e}{\Gamma} = (17.82 \pm 0.04)\%, \qquad \frac{\Gamma_\mu}{\Gamma} = (17.39 \pm 0.04)\%,$$

where Γ is the total decay width of τ lepton. Using these numbers I get that

$$R_\tau = 1.0247 \pm 0.0033. \tag{6.16}$$

The SM expression for R_τ including radiative corrections is [7]

$$R_\tau^{SM} = \frac{f(m_e^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)} = 1.028, \tag{6.17}$$

where $f(x)$ is already defined in eq. (6.9). Eq.s (6.16) and (6.17) then result in

$$\frac{R_\tau}{R_\tau^{SM}} = 0.9968 \pm 0.0032.$$

Finally inserting the above number in eq. (6.15) yields the following 2σ level bounds:

$$|\lambda_{23k}| < 0.06\left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 2\sigma \tag{6.18}$$

$$|\lambda_{13k}| < 0.03\left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right). \qquad 2\sigma \tag{6.19}$$

6.2 Charged Current Universality in π and τ Decays

One can also consider the π leptonic decays in $\not{R}_P$ SUSY as they can be mediated by the $LQ\overline{d}$ operators at the tree level through vertex ($\not{R}_P 4$) and the counterpart of vertex ($\not{R}_P 7$) with all the arrows reversed as shown in Fig. 8.

Diagram 8 (a) yields the following tree level $\not{R}_P$ matrix element at low energies:

$$\begin{aligned} i\mathcal{M}^{\not{R}_P} &= \left\{x_1^{\alpha}\left[-i\lambda'_{i1k}\,\delta_{\alpha}{}^{\beta}\right]y_{3\beta}\right\}\left(\frac{i}{t-m^2_{\widetilde{d}_{Rk}}}\right)\left\{y^{\dagger}_{2\dot{\alpha}}\left[i\lambda'^{*}_{i1k}\,\delta^{\dot{\alpha}}{}_{\dot{\beta}}\right]x_4^{\dagger\dot{\beta}}\right\} \\ &= \frac{i\,|\lambda'_{i1k}|^2}{\left(t-m^2_{\widetilde{d}_{Rk}}\right)}\{x_1y_3\}\{y_2^{\dagger}x_4^{\dagger}\} \\ &\overset{t\ll m^2_{\widetilde{d}}}{\approx} -\frac{i\,|\lambda'_{i1k}|^2}{m^2_{\widetilde{d}_{Rk}}}\{x_1y_3\}\{y_2^{\dagger}x_4^{\dagger}\} \\ &\overset{(2.16)}{=} \frac{i\,|\lambda'_{i1k}|^2}{2m^2_{\widetilde{d}_{Rk}}}\{x_1\sigma^{\mu}y_2^{\dagger}\}\{x_4^{\dagger}\overline{\sigma}_{\mu}y_3\}. \end{aligned} \tag{6.20}$$

Using the vertex factors (S6) and (S7), I get the following SM matrix element at low energies:

$$\mathcal{M}^{SM} = \frac{g^2V_{ud}}{2M_W^2}\{x_1\sigma^{\mu}y_2^{\dagger}\}\{x_4^{\dagger}\overline{\sigma}_{\mu}y_3\}. \tag{6.21}$$

There is no relative minus sign between the diagrams, so we get a constructive contribution from the $\not{R}_p$ operator:

$$\begin{aligned} \mathcal{M}^{Total} \propto& \frac{V_{ud}}{2}\left(\frac{g^2}{M_W^2}\right)\left[1+\frac{1}{V_{ud}}\left(\frac{M_W^2}{g^2}\frac{|\lambda'_{i1k}|^2}{m^2_{\widetilde{d}_{Rk}}}\right)\right] \\ =& \frac{V_{ud}}{2}\left(\frac{g^2}{M_W^2}\right)\left[1+\frac{1}{V_{ud}}r'_{i1k}(\widetilde{d}_{Rk})\right]. \end{aligned} \tag{6.22}$$

The above result can be used to extract bounds on λ'_{11k} and λ'_{21k} by comparing the ratio $R_\pi \equiv \dfrac{\Gamma(\pi^- \to e^-\overline{\nu}_e)}{\Gamma(\pi^- \to \mu^-\overline{\nu}_\mu)}$ [4] to the experimental value. The advantage of the ratio is that it eliminates the dependence on the pion decay coupling constant, f_π. By using eq. (6.22), I get the following expression for R_π (the calculation is similar to eq. (6.6)):

$$R_\pi \approx R_\pi^{SM}\left[1+\frac{2}{V_{ud}}\left(r'_{11k}(\widetilde{d}_{Rk})-r'_{21k}(\widetilde{d}_{Rk})\right)\right]. \tag{6.23}$$

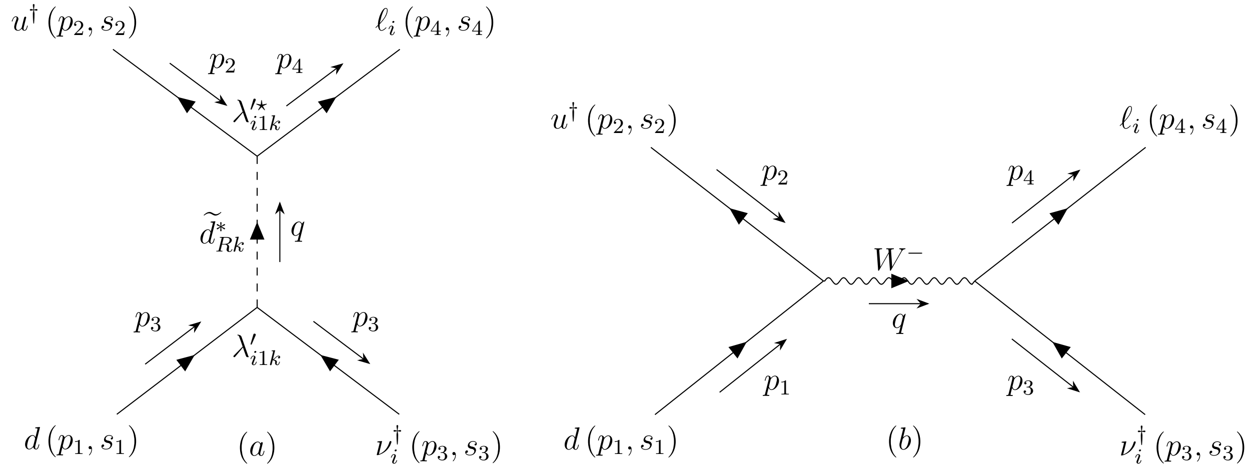


Figure 8: Pion leptonic decay in (a) an $\not{R}_p$ SUSY and (b) the Standard Model. $q^2 = t = (p_1 - p_3)^2$ in (a), while $q^2 = s = (p_1 + p_2)^2$ in (b).

The tree-level SM expression for R_π is [8]

$$R^{SM}_{\pi,\,\text{tree}} = \left(\frac{m_e}{m_\mu}\right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right)^2. \tag{6.24}$$

After plugging $m_e \approx 0.511$ MeV, $m_\mu \approx 105.6584$ MeV and $m_{\pi^\pm} \approx 139.5706 \pm 0.0002$ MeV [6] in eq. (6.24), I get that

$$R^{SM}_{\pi,\,\text{tree}} = 1.284 \times 10^{-4}. \tag{6.25}$$

With the radiative corrections $R^{SM}_{\pi,\,\text{tree}}$ becomes [9]

$$R^{SM}_{\pi} = R^{SM}_{\pi,\,\text{tree}}(1 + \delta R_\pi) = 1.236 \times 10^{-4}, \tag{6.26}$$

with [9]

$$\delta R_\pi = -0.0374 \pm 0.0001. \tag{6.27}$$

The measured branching ratios are [6]

$$\pi \to e\nu_e \qquad (\Gamma_e/\Gamma) = (1.230 \pm 0.004) \times 10^{-4},$$
$$\pi \to \mu\nu_\mu \qquad (\Gamma_\mu/\Gamma) = (99.98770 \pm 0.00004) \times 10^{-2},$$

where Γ is a total decay width of $\pi^\pm$. Calculating R_π using the above values, together with eq. (6.26), yields the following value for the ratio:

$$\frac{R_\pi}{R^{SM}_\pi} = 0.9951 \pm 0.0032. \tag{6.28}$$

Finally, after inserting eq. (6.28) into eq. (6.23), I get the following single bounds:

$$|\lambda'_{21k}| = 0.04 \begin{array}{l} +0.01 \\ -0.02 \end{array} (< 0.06) \left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma) \tag{6.29}$$

$$|\lambda'_{11k}| < 0.02 \left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right). \qquad 2\sigma\,(\text{excluded at } 1\sigma \text{ level}) \tag{6.30}$$

The 2-body decay $\tau^- \to \pi^- \nu_\tau$, shown in Fig. 9, can also be used as an additional test of lepton universality.

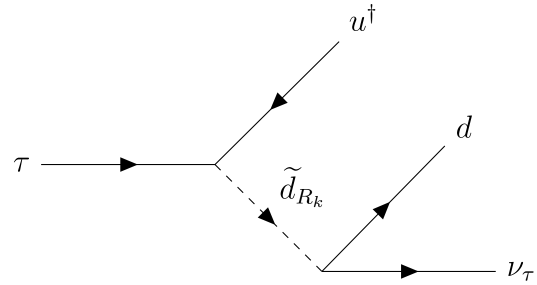


Figure 9: $\not{R}_P$ contribution to the tau decay.

Once again, comparing the ratio [3]

$$R_{\tau\pi} \equiv \frac{\Gamma(\tau^- \to \pi^- \overline{\nu}_\tau)}{\Gamma(\pi^- \to \mu^- \nu_\mu)} \approx R^{SM}_{\tau\pi}\left[1 + \frac{2}{V_{ud}}\,(r'_{31k} - r'_{21k})\right] \tag{6.31}$$

to the experimental result will yield bounds on the couplings λ'_{31k} and λ'_{21k}.

The tree-level SM expression for the above ratio is [10]

$$R^{SM}_{\tau\pi,\,\mathrm{tree}} = \frac{m_\tau^3}{2m_\pi m_\mu^2}\frac{(1 - m_\pi^2/m_\tau^2)^2}{(1 - m_\mu^2/m_\pi^2)^2}. \tag{6.32}$$

$m_\tau = 1776.86 \pm 0.12$ MeV, τ_τ(lifetime) $= (290.3 \pm 0.5) \times 10^{-15}$s, $\tau_{\pi^\pm} = (2.6033 \pm 0.0005) \times 10^{-8}$s and the branching ratio for $\tau \to \pi\nu_\tau$ is $(\Gamma_i/\Gamma) = (10.82 \pm 0.05)\%$ [6]. After plugging these numbers into eq. (6.32) I obtain that

$$R^{SM}_{\tau\pi,\,\mathrm{tree}} = 0.9756 \times 10^4. \tag{6.33}$$

$R^{SM}_{\tau\pi,\,\mathrm{tree}}$ is modified by the radiative corrections to [9]

$$R_{\tau\pi} = R^{SM}_{\tau\pi,\,\mathrm{tree}}(1 + \delta R_{\tau\pi}) \left(9.771 \begin{array}{l} +0.009 \\ -0.013 \end{array}\right) \times 10^3, \tag{6.34}$$

with [9]

$$\delta R_{\tau\pi} = 0.0016 \begin{array}{l} +0.0009 \\ -0.0014 \end{array}. \tag{6.35}$$

Experimental value of $R_{\tau\pi}$ is

$$R_{\tau\pi} = (9.711 \pm 0.052) \times 10^3,$$

so that I calculate the ratio to be

$$\frac{R_{\tau\pi}}{R_{\tau\pi}^{SM}} = 0.9939 \pm 0.0054. \tag{6.36}$$

Finally, after inserting the above result into eq. (6.31), I get the following single bounds:

$$|\lambda'_{21k}| = 0.04 \begin{array}{l} +0.02 \\ -0.03 \end{array} \ (< 0.07) \left(\frac{m_{\tilde{d}_{Rk}}}{100\,\mathrm{GeV}} \right), \qquad 1\sigma\ (2\sigma) \tag{6.37}$$

$$|\lambda'_{31k}| < 0.04 \left(\frac{m_{\tilde{d}_{Rk}}}{100\,\mathrm{GeV}} \right). \qquad 2\sigma\ (\text{excluded at } 1\sigma \text{ level}) \tag{6.38}$$

6.3 Charged Current Universality in the Quark Sector

In the quark sector the $LQ\bar{d}$ operators lead to extra contributions to quark semileptonic decays, as happens, for instance, in nuclear β decay. The form of these decays are indeed similar to that of pion leptonic decay, diagrammatically shown in Fig. 8(a), with the incoming up anti-quark external line reversed and the coupling set to λ'_{11k}. The calculation of the $\not{R}_p$ contribution is very similar to that of eq. (6.4). This implies that, at low energies, the effective tree-level weak coupling now is:

$$\frac{g^2}{8M_W^2} \left[V_{ud}^{SM} + r'_{11k}(\tilde{d}_{Rk}) \right]. \tag{6.39}$$

The CKM matrix elements V_{Qq} are experimentally determined from the ratio of the $Q \to q e \nu_e$ to $\mu \to \nu_\mu e \nu_e$ partial widths [4], which for $Q = d$ and $q = u$ is

$$|V_{ud}|^2_{expt} = \frac{|V_{ud}^{SM} + r'_{11k}(\tilde{d}_{Rk})|^2}{|1 + r_{12k}(\tilde{\ell}_{Rk})|^2}. \tag{6.40}$$

The rates for $s \to u e \nu_e$ and $b \to u e \nu_e$ will also be modified by the $LQ\bar{d}$ interactions [3]. One should keep in mind that these rates will still depend on r_{12k} through the dependence of G_F on the λ_{12k} couplings. Summing over all of the down-type quark generations yields:

$$\begin{aligned} \sum_{j=1}^{3} |V_{ud_j}^2|_{expt} &= \frac{1}{|1 + r_{12k}(\tilde{\ell}_{Rk})|^2} \Big[|V_{ud}^{SM} + r'_{11k}(\tilde{d}_{Rk})|^2 + |V_{us}^{SM} + \{r'_{11k}(\tilde{d}_{Rk}) r'_{12k}(\tilde{d}_{Rk})\}^{1/2}|^2 \\ &\quad + |V_{ub}^{SM} + \{r'_{11k}(\tilde{d}_{Rk}) r'_{13k}(\tilde{d}_{Rk})\}^{1/2}|^2 \Big] \\ &\approx |V_{ud}^{SM}|^2 \left[1 + \frac{2}{V_{ud}^{SM}} r'_{11k} - 2\, r_{12k} \right] + |V_{us}^{SM}|^2 \left[1 + \frac{2}{V_{us}^{SM}} (r'_{11k} r'_{12k})^{1/2} - 2\, r_{12k} \right] \end{aligned}$$

$$+ |V_{ub}^{SM}|^2 \left[1 + \frac{2}{V_{ub}^{SM}} (r'_{11k} r'_{13k})^{1/2} - 2\, r_{12k}\right]$$
$$= 1 - 2\, r_{12k}(\widetilde{\ell}_{Rk}) + 2V_{ud}^{SM} r'_{11k}(\widetilde{d}_{Rk})$$
$$+ 2V_{us}^{SM} \left[r'_{11k}(\widetilde{d}_{Rk}) r'_{12k}(\widetilde{d}_{Rk})\right]^{1/2} + 2V_{ub}^{SM} \left[r'_{11k}(\widetilde{d}_{Rk}) r'_{13k}(\widetilde{d}_{Rk})\right]^{1/2}, \qquad (6.41)$$

where the unitarity of the CKM matrix has been used in the last step.

The numerical value of the above quantity is [6]

$$\sum_{j=1}^{3} |V_{ud_j}^2|_{expt} = 0.9994 \pm 0.0005.$$

Also, at the lowest order in $\not{R}_p$ corrections we can take [3] $V_{ud}^{SM} = V_{ud}^{expt} \approx 0.974$ [6]. After plugging these two numbers into eq. (6.41), I get the following single bounds:

$$|\lambda_{12k}| = 0.014 \begin{array}{l} +0.005 \\ -0.008 \end{array} \; (< 0.023) \left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right) \qquad 1\sigma\ (2\sigma), \qquad (6.42)$$

$$|\lambda'_{11k}| < 0.011 \left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) \qquad 2\sigma \text{ (it is excluded at } 1\sigma \text{ level).} \qquad (6.43)$$

6.4 Semileptonic and Leptonic Decays of D Mesons

Since the experimental branching ratios for the three classes of semileptonic charmed meson decays are available, I will now discuss the following three-body decay channels: $D^+ \to \overline{K}^0 \ell_i^+ \nu_i$, $D^+ \to \overline{K}^{0*} \ell_i^+ \nu_i$, $D^0 \to \overline{K}^- \ell_i^+ \nu_i$, where $i = e, \mu$. Apart from the normal Standard Model diagrams, we get additional contributions to these decays due to the λ'_{i2k} couplings of the $LQ\overline{d}$ Lagrangian. The tree-level $\not{R}_P$ Feynman diagram of the $D^+ \to \overline{K}^0 \ell_i^+ \nu_i$ decay is shown in Fig. 10 (a) [3].

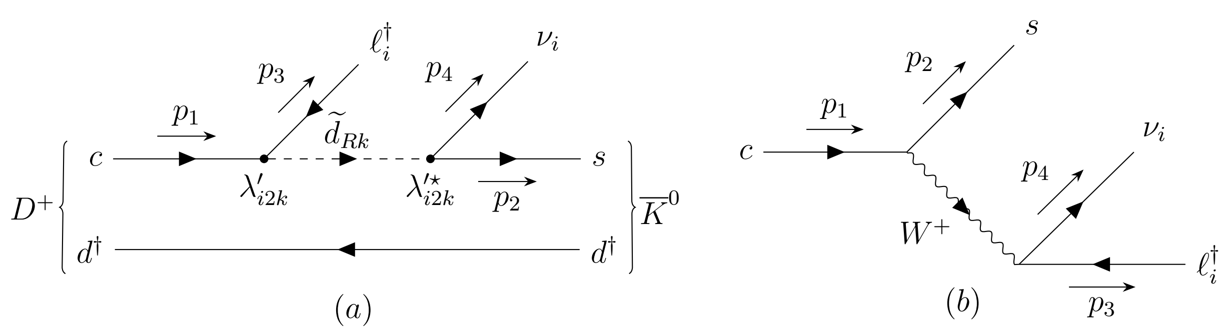


Figure 10: (a) $\not{R}_P$ and (b) SM contributions to the D^+ meson semileptonic decay.

Using the vertex rules $(\not{R}_P 7)$ and the complex conjugate of $(\not{R}_P 4)$ the tree-level $\not{R}_P$ matrix element is then:

$$
\begin{aligned}
i\mathcal{M}^{\not{R}_P} &= \left\{x_1^{\alpha}\left[i\lambda'_{i2k}\,\delta_{\alpha}{}^{\beta}\right]y_{3\beta}\right\}\left(\frac{i}{t-m^2_{\tilde{d}_{Rk}}}\right)\left\{x^{\dagger}_{2\dot{\alpha}}\left[-i\lambda'^{*}_{i2k}\,\delta^{\dot{\alpha}}{}_{\dot{\beta}}\right]x_4^{\dagger\dot{\beta}}\right\}\\
&= \frac{i\,|\lambda'_{i2k}|^2}{(t-m^2_{\tilde{d}_{Rk}})}\,\{x_1y_3\}\,\{x_2^{\dagger}x_4^{\dagger}\}\\
&\overset{s\ll m^2_{\tilde{d}}}{\approx} -\frac{i\,|\lambda'_{i2k}|^2}{m^2_{\tilde{d}_{Rk}}}\,\{x_1y_3\}\,\{x_2^{\dagger}x_4^{\dagger}\}\\
&\overset{(2.16)}{=} \frac{i\,|\lambda'_{i2k}|^2}{2m^2_{\tilde{d}_{Rk}}}\,\{x_1\sigma^{\mu}x_2^{\dagger}\}\{x_4^{\dagger}\overline{\sigma}_{\mu}y_3\}.
\end{aligned}
\tag{6.44}
$$

The SM Feynman diagram involves the vertices (S5) and (S8) and yields the following tree-level matrix element (with 1234 ordering) at low energies:

$$
\mathcal{M}^{SM} \approx \frac{-V_{cs}g^2}{2M_W^2}\,\{x_1\sigma^{\mu}x_2^{\dagger}\}\{x_4^{\dagger}\overline{\sigma}_{\mu}y_3\}. \tag{6.45}
$$

Since there is a relative minus sign due to the ordering of the external fermions between the two diagrams, we get a constructive interference for the effective coupling: $\dfrac{V_{cs}g^2}{2M_W^2}\left[1+\dfrac{r'_{i2k}}{V_{cs}}\right]$.

The other decays are identical, so after defining [3] $R^{(*)}_{D^+}, R_{D^0} \equiv B(D\to\mu\nu_{\mu}K^{(*)})/B(D\to e\nu_e K^{(*)})$ respectively, the $\not{R}_P$ contributions car be rewritten as

$$
\frac{R_{D^+}}{(R_{D^+})^{SM}} = \frac{R^*_{D^+}}{(R^*_{D^+})^{SM}} = \frac{R_{D^0}}{(R_{D^0})^{SM}} = \frac{|V_{cs}+r'_{22k}(\widetilde{d}_{Rk})|^2}{|V_{cs}+r'_{12k}(\widetilde{d}_{Rk})|^2} \approx 1+\frac{2}{|V_{cs}|}\left\{r'_{22k}(\widetilde{d}_{Rk})-r'_{12k}(\widetilde{d}_{Rk})\right\}. \tag{6.46}
$$

The measured branching ratios for the different decay channels are the following [6]

$$
\begin{aligned}
&D^+\to\overline{K}^0e^+\nu_e && (\Gamma_i/\Gamma)=(8.73\pm0.10)\,\%,\\
&D^+\to\overline{K}^0\mu^+\nu_{\mu} && (\Gamma_i/\Gamma)=(8.76\pm0.19)\,\%,\\
&D^+\to\overline{K}^*(892)^0e^+\nu_e && (\Gamma_i/\Gamma)=(5.40\pm0.10)\,\%,\\
&D^+\to\overline{K}^*(892)^0\mu^+\nu_{\mu} && (\Gamma_i/\Gamma)=(5.27\pm0.15)\,\%,\\
&D^0\to K^-e^+\nu_e && (\Gamma_i/\Gamma)=(3.542\pm0.035)\,\%,\\
&D^0\to K^-\mu^+\nu_{\mu} && (\Gamma_i/\Gamma)=(3.41\pm0.04)\,\%,
\end{aligned}
$$

where Γ is the total decay width of $D^{+(0)}$ meson. By using the above numbers I calculate that

$$
R_{D^+} = 1.003\pm0.025, \tag{6.47}
$$

$$R^*_{D^+} = 0.976 \pm 0.033, \tag{6.48}$$

$$R_{D^0} = 0.9627 \pm 0.0148. \tag{6.49}$$

$(R_D^{(*)})^{SM} = 1/1.03$ accounts for the phase space suppression in the muon channel [3]. This would be the exact value of R_{D^+} and R_{D^0} if one treats the involved mesons as point particles [7]. Ref. [7] takes into account the form factors and calculates R^{SM} for all three cases based on experimental date. For $R^{SM}_{D^+}$ and $R^{SM}_{D^0}$, the above value of $1/1.03$ is a good approximation, since it lies within the calculated range [7]. But for $R^{*SM}_{D^+}$ we have to include form factors, since $1/1.03$ is out of the calculated range [7]

$$\left[R^{*SM}_{D^+}\right]^{-1} = 1.060 \begin{array}{l} +0.005 \\ -0.007 \end{array} . \qquad (2\sigma) \tag{6.50}$$

Using eq. (6.50) I get that the ratio of the experimental and theoretical values of $R^*_{D^+}$ is

$$\frac{R^*_{D^+}}{R^{*SM}_{D^+}} = 1.0346 \pm 0.0363. \tag{6.51}$$

$|V_{cs}| \approx 0.997$ [6]. This value together with equations (6.46) - (6.51) then yield the following single bounds:

$$R_{D^+} \begin{cases} |\lambda'_{22k}| < 0.16 \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 2\sigma, \\ |\lambda'_{12k}| < 0.08 \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 2\sigma \text{ (it is excluded at } 1\sigma \text{ level)}, \end{cases} \tag{6.52}$$

$$R^*_{D^+} \begin{cases} |\lambda'_{22k}| < 0.15\,(0.18) \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 1\sigma\ (2\sigma), \\ |\lambda'_{12k}| < 0.02\,(0.11) \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 1\sigma\ (2\sigma), \end{cases} \tag{6.53}$$

$$R_{D^0} \begin{cases} |\lambda'_{22k}| < 0.05\,(0.08) \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 1\sigma\ (2\sigma), \\ |\lambda'_{12k}| < 0.09\,(0.11) \left(\dfrac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right) & 1\sigma\ (2\sigma). \end{cases} \tag{6.54}$$

Another useful process for testing the lepton universality is the two-body leptonic decay of the strange D_s meson: $D_s^- \to \ell^- \overline{v}_\ell$. The $\not{R}_P$ diagram is is shown in Fig. 11.

Additional single-coupling constant bounds can be obtained by defining $R_{D_s}(\tau\mu) \equiv B(D_s \to \tau\nu_\tau)/B(D_s \to \mu\nu_\mu)$ [3] as the ratio of τ and μ decay channels. $\not{R}_P$ couplings contribute to this ratio, giving (calculation is analogous to the R^*_D case):

$$\frac{R_{D_s}(\tau\mu)}{R^{SM}_{D_s}(\tau\mu)} = \frac{|V_{cs} + r'_{32k}(\widetilde{d}_{Rk})|^2}{|V_{cs} + r'_{22k}(\widetilde{d}_{Rk})|^2} \approx 1 + \frac{2}{|V_{cs}|}\left\{r'_{32k}(\widetilde{d}_{Rk}) - r'_{22k}(\widetilde{d}_{Rk})\right\}. \tag{6.55}$$

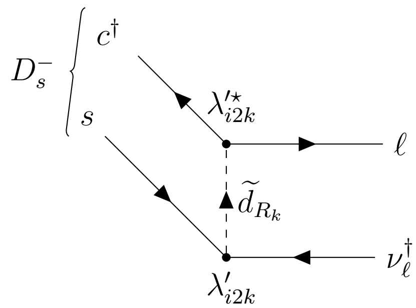


Figure 11: $\not{R}_P$ contributions to the D_s^- meson leptonic decay.

The measured branching ratios are the following [6]:

$$D_s \to \tau\nu_\tau \qquad (\Gamma_i/\Gamma) = (5.48 \pm 0.23)\,\%,$$
$$D_s \to \mu\nu_\mu \qquad (\Gamma_i/\Gamma) = (5.50 \pm 0.23) \times 10^{-3}.$$

The SM expression of $R^{SM}_{D_s}(\tau\mu)$ is [11]

$$R^{SM}_{D_s}(\tau\mu) = \left(\frac{m_\tau}{m_\mu}\right)^2 \left(\frac{M^2_{D_s} - m^2_\tau}{M^2_{D_s} - m^2_\mu}\right)^2. \tag{6.56}$$

Using numerical values of the masses involved in eq. (6.56) [6]

$$M_{D_s^\pm} = 1968.34 \pm 0.07\,\text{MeV}, \qquad m_\mu \approx 105.658\,\text{MeV}, \qquad m_\tau \approx 1777\,\text{MeV},$$

and the above numbers for the branching ratios, I get that

$$R^{SM}_{D_s}(\tau\mu) \approx 9.73, \qquad R_{D_s}(\tau\mu) = 9.96 \pm 0.59. \tag{6.57}$$

Finally plugging eq. (6.57) in eq. (6.55) gives me the following bounds:

$$|\lambda'_{22k}| < 0.11\,(0.18)\left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\text{GeV}}\right) \qquad 1\sigma\,(2\sigma), \tag{6.58}$$
$$|\lambda'_{32k}| < 0.16\,(0.21)\left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\text{GeV}}\right) \qquad 1\sigma\,(2\sigma). \tag{6.59}$$

6.5 Neutrino Interactions

In this section I discuss neutrino interactions with leptons and nucleons. I will focus on muon neutrino - electron elastic scattering $\nu_\mu(\overline{\nu}_\mu)e^- \to \nu_\mu(\overline{\nu}_\mu)e^-$ and NC muon neutrino - nucleon deep inelastic scatterings $\nu_\mu(\overline{\nu}_\mu)N(A) \to \nu_\mu(\overline{\nu}_\mu)X$, where X represents any product particle.

At low energies the effective Lagrangian containing the relevant neutral current couplings encoded in the parameters $g^{\nu\ell}_{L,R}$ and $\epsilon_{L,R}(q)$ for the charged leptons and the quarks, respectively, is [3]:

$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}}\;(\nu^\dagger\overline{\sigma}_\mu\nu)\left\{\sum_{\ell=e,\mu} g^{\nu\ell}_L\left(\ell^\dagger\overline{\sigma}^\mu\ell\right) + g^{\nu\ell}_R\left(\overline{\ell}\sigma^\mu\overline{\ell}^\dagger\right) + \sum_{q=u,d}\epsilon_L(q)\left(q^\dagger\overline{\sigma}^\mu q\right) + \epsilon_R(q)\left(\overline{q}\sigma^\mu\overline{q}^\dagger\right)\right\}. \tag{6.60}$$

In the presence of the $\not{R}_p$ interactions the above-mentioned neutrino processes get additional tree-level contributions, consequently modifying the effective NC couplings. Measuring them can thus provide single coupling bounds on the involved $\not{R}_p$ coulings.

First let's discuss $\nu_\mu e^-$ scattering. The relevant $\not{R}_p$ diagram, shown in Fig. 12 (b), is obtained through vertex ($\not{R}_p$1) and its counterpart with all the arrows reversed.

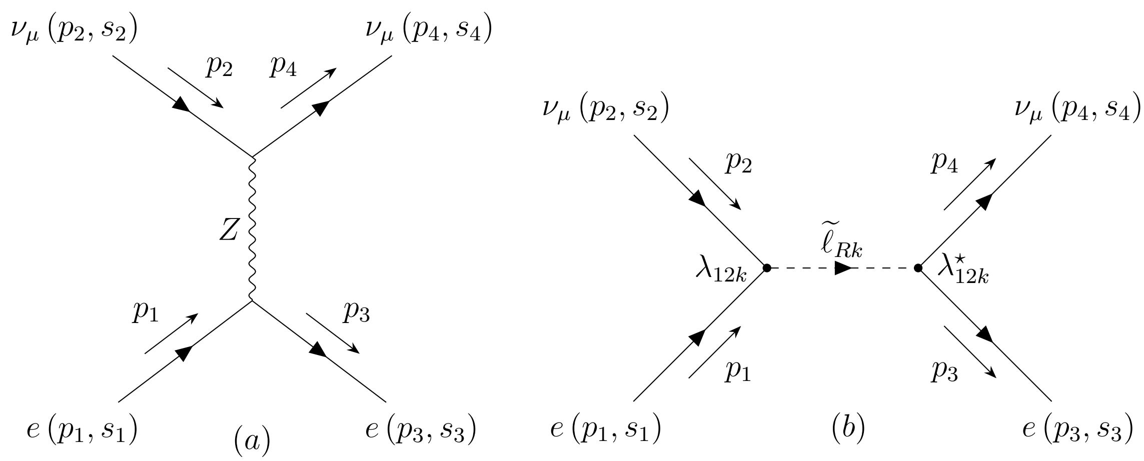


Figure 12: $\nu_\mu e^-$ elastic scattering in (a) the SM and (b) an $\not{R}_p$ SUSY.

The above diagrams yield the following matrix elements:

$$\begin{aligned} i\mathcal{M}^{\not{R}_P} &= \{x_1\,[i\lambda_{12k}\,]\,x_2\}\left(\frac{i}{s-m^2_{\widetilde{\ell}_{Rk}}}\right)\{x_3^\dagger[i\lambda^*_{12k}]x_4^\dagger\} \\ &= \frac{-i\,|\lambda_{12k}|^2}{s-m^2_{\widetilde{\ell}_{Rk}}}\,\{x_1x_2\}\,\{x_3^\dagger x_4^\dagger\} \\ &\overset{s\,\ll\, m^2_{\widetilde{\ell}}}{\approx} \frac{i\,|\lambda_{12k}|^2}{m^2_{\widetilde{\ell}_{Rk}}}\,\{x_1x_2\}\,\{x_3^\dagger x_4^\dagger\} \\ &\overset{(2.17)}{=} -\frac{i\,|\lambda_{12k}|^2}{2m^2_{\widetilde{\ell}_{Rk}}}\,\{x_3^\dagger\overline{\sigma}^\mu x_1\}\{x_4^\dagger\overline{\sigma}_\mu x_2\}. \end{aligned} \tag{6.61}$$

$$i\mathcal{M}^{SM} = \left\{x_1\left(\frac{ig}{c_W}\left[-\frac{1}{2}+s^2_W\right]\sigma_\mu\right)x_3^\dagger\right\}\left[\frac{-ig^{\mu\nu}}{t-M_Z^2}\right]\left\{x_2\left(\frac{ig}{c_W}\left[\frac{1}{2}\right]\sigma_\nu\right)x_4^\dagger\right\}$$

$$= \frac{i(g^2/c_W^2)}{t - M_Z^2}\left[\frac{1}{2}\right]\left[-\frac{1}{2}+s_W^2\right]\{x_3^\dagger\overline{\sigma}^\mu x_1\}\{x_4^\dagger\overline{\sigma}_\mu x_2\}$$
$$\overset{t\ll M_Z^2}{\approx} -i\left[\frac{1}{2}\right]\left[-\frac{1}{2}+s_W^2\right]\left(\frac{g^2}{M_W^2}\right)\{x_3^\dagger\overline{\sigma}^\mu x_1\}\{x_4^\dagger\overline{\sigma}_\mu x_2\}. \tag{6.62}$$

Now taking into account that there is a relative minus sign between the diagrams due to the external fermion ordering, we get the destructive interference between the couplings:

$$-\frac{1}{2}\left(\frac{g^2}{M_W^2}\right)\left(-\frac{1}{2}+s_W^2\right)+\frac{1}{2}\frac{|\lambda|^2}{m^2_{\widetilde{\ell}_{Rk}}} = -\frac{1}{2}\left(\frac{g^2}{M_W^2}\right)\left[\left(-\frac{1}{2}+s_W^2\right)-\left(\frac{M_W^2}{g^2}\right)\frac{|\lambda|^2}{m^2_{\widetilde{\ell}_{Rk}}}\right]$$
$$= -\frac{1}{2}\left(\frac{g^2}{M_W^2}\right)\left[\left(-\frac{1}{2}+s_W^2\right)-r_{12k}(\widetilde{\ell}_{Rk})\right] = -\frac{4G_F}{\sqrt{2}}\left\{1-r_{12k}(\widetilde{\ell}_{Rk})\right\}\left[\left(-\frac{1}{2}+s_W^2\right)-r_{12k}(\widetilde{\ell}_{Rk})\right].$$

$$\Longrightarrow\; g_L^{\nu e} \approx \left(-\frac{1}{2}+s_W^2\right)-\left(-\frac{1}{2}+s_W^2\right)r_{12k}(\widetilde{\ell}_{Rk})-r_{12k}(\widetilde{\ell}_{Rk})$$
$$= \left(-\frac{1}{2}+s_W^2\right)\left[1-r_{12k}(\widetilde{\ell}_{Rk})\right]-r_{12k}(\widetilde{\ell}_{Rk}).$$

$\overline{\nu}_\mu e^-$ scattering also receives $\not{R}_p$ contribution at tree-level, this time through vertex ($\not{R}_p$3). The relevant Feynman diagram is shown in Fig. 13 (b) below.

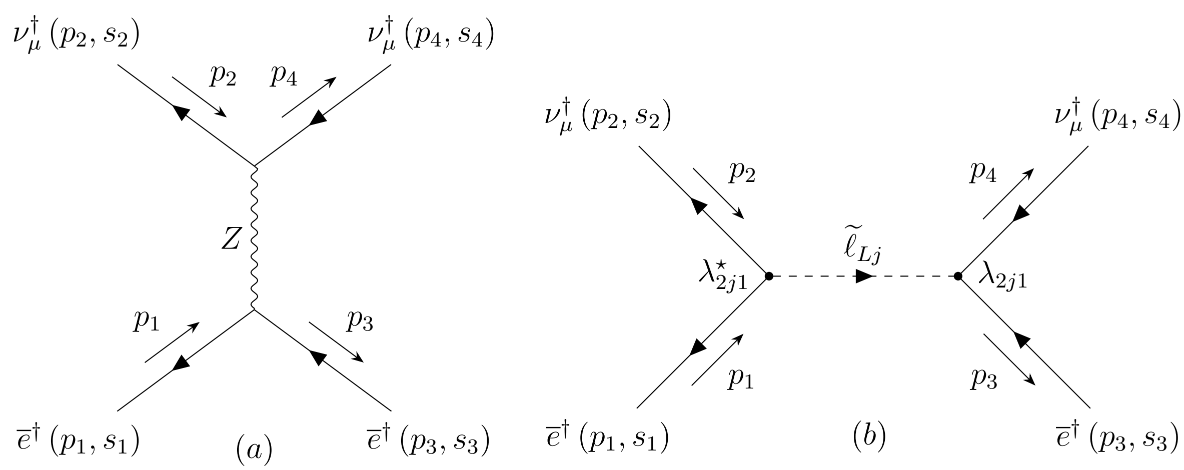


Figure 13: $\overline{\nu}_\mu e^-$ elastic scattering in (a) the SM and (b) an $\not{R}_p$ SUSY.

At low energies the matrix elements (the calculation is similar to the $\nu_\mu e^-$ case) are:

$$\mathcal{M}^{SM} = \frac{1}{2}\left(\frac{g^2}{M_W^2}\right)s_W^2\,\{y_1^\dagger\overline{\sigma}^\mu y_3\}\{y_2^\dagger\overline{\sigma}_\mu y_4\}. \tag{6.63}$$
$$\mathcal{M}^{\not{R}_P} = -\frac{1}{2}\frac{|\lambda_{2j1}|^2}{m^2_{\widetilde{\ell}_{Lj}}}\{y_1^\dagger\overline{\sigma}^\mu y_3\}\{y_2^\dagger\overline{\sigma}_\mu y_4\}. \tag{6.64}$$

For the effective coupling it means:

$$-\frac{1}{2}\left(\frac{g^2}{M_W^2}\right)s_W^2 - \frac{1}{2}\frac{|\lambda_{2j1}|^2}{m^2_{\widetilde{\ell}_{Lj}}} = -\frac{1}{2}\left(\frac{g^2}{M_W^2}\right)\left[s_W^2 + r_{2j1}(\widetilde{\ell}_{Lj})\right] = -\frac{4G_F}{\sqrt{2}}\left[\{1 - r_{12k}\}\, s_W^2 + r_{2j1}\right].$$

$$\Longrightarrow\ g_R^{\nu e} = s_W^2\left[1 - r_{12k}(\widetilde{\ell}_{Rk})\right] + r_{211}(\widetilde{\ell}_{L1}) + r_{231}(\widetilde{\ell}_{L3}).$$

As for the neutrino-nucleon deep inelastic scattering, represented by $\nu_\mu(\overline{\nu}_\mu)d \to \nu_\mu(\overline{\nu}_\mu)d$ processes at the quark level, the diagrams (and hence matrix elements) are obtained by applying the following changes: $e \to d$, $\lambda_{12k} \to \lambda'_{21k}$, $\widetilde{\ell}_{Rk} \to \widetilde{d}_{Rk}$ and $e \to d$, $\lambda_{2j1} \to \lambda'_{2j1}$, $\widetilde{\ell}_{Lj} \to \widetilde{d}_{Lj}$ in Fig.s 12 (b) and Fig. 13 (b), respectively. Thus we get similar expressions for the NC couplings $\epsilon_L(d)$ and $\epsilon_R(d)$ but with appropriate factors and λ couplings. Altogether we have:

$$g_L^{\nu e} = (-\frac{1}{2} + s_W^2)[1 - r_{12k}(\widetilde{\ell}_{Rk})] - r_{12k}(\widetilde{\ell}_{Rk}), \tag{6.65}$$

$$g_R^{\nu e} = s_W^2\left[1 - r_{12k}(\widetilde{\ell}_{Rk})\right] + r_{211}(\widetilde{\ell}_{L1}) + r_{231}(\widetilde{\ell}_{L3}), \tag{6.66}$$

$$\epsilon_L(d) = (-\frac{1}{2} + \frac{1}{3}s_W^2)[1 - r_{12k}(\widetilde{\ell}_{Rk})] - r'_{21k}(\widetilde{d}_{Rk}), \tag{6.67}$$

$$\epsilon_R(d) = \frac{s_W^2}{3}\left[1 - r_{12k}(\widetilde{\ell}_{Rk})\right] + r'_{2j1}(\widetilde{d}_{Lj}). \tag{6.68}$$

In order to set bounds on the coupling constants involved notice that [6]

$$g_L^{\nu e} = \frac{1}{2}(g_V^{\nu e} + g_A^{\nu e}), \tag{6.69}$$

$$g_R^{\nu e} = \frac{1}{2}(g_V^{\nu e} - g_A^{\nu e}), \tag{6.70}$$

where

$$g_V^{\nu e} = 2g_V^{\nu} g_V^{e}, \tag{6.71}$$

$$g_A^{\nu e} = 2g_A^{\nu} g_A^{e}, \tag{6.72}$$

and

$$g_V^f = T_3^f - 2s_W^2 Q_f, \tag{6.73}$$

$$g_A^f = T_3^f. \tag{6.74}$$

The experimental and SM values (including radiative corrections) for $g_V^{\nu e}$ and $g_A^{\nu e}$ are [6]

$$g_V^{\nu e} = -0.040 \pm 0.015, \qquad (g_V^{\nu e})^{\rm SM} = -0.0398 \pm 0.0001, \tag{6.75}$$

$$g_A^{\nu e} = -0.507 \pm 0.014, \qquad (g_A^{\nu e})^{\rm SM} = -0.5064. \tag{6.76}$$

Using eq.s (6.75) and (6.76) I find that

$$g_L^{\nu e} = -0.2735 \pm 0.0103, \qquad (g_L^{\nu e})^{\rm SM} = -0.2731, \tag{6.77}$$
$$g_R^{\nu e} = 0.2335 \pm 0.0103, \qquad (g_R^{\nu e})^{\rm SM} = 0.2333. \tag{6.78}$$

The experimental and SM values (including radiative corrections) for $\epsilon_L(d)$ and $\epsilon_R(d)$ are [6]

$$\epsilon_L(d) = -0.4554 \pm 0.0340, \qquad (\epsilon_L(d))^{\rm SM} = -0.4288, \tag{6.79}$$
$$\epsilon_R(d) = 0.0738 \pm 0.0340, \qquad (\epsilon_R(d))^{\rm SM} = 0.0777. \tag{6.80}$$

Finally, the numbers in eq.s (6.77) - (6.80) give me the following single bounds:

$$|\lambda_{12k}| < 0.097\,(0.136)\left(\frac{m_{\tilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [g_L^{\nu e}] \tag{6.81}$$
$$|\lambda_{231}| < 0.082\,(0.115)\left(\frac{m_{\tilde{\tau}_L}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [g_R^{\nu e}] \tag{6.82}$$
$$|\lambda_{121}| < 0.093\,(0.131)\left(\frac{m_{\tilde{e}_L}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [g_R^{\nu e}] \tag{6.83}$$
$$|\lambda_{12k}| < 0.166\,(0.236)\left(\frac{m_{\tilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad k\neq 1\;\; 1\sigma\,(2\sigma)\quad [g_R^{\nu e}] \tag{6.84}$$
$$|\lambda_{12k}| < 0.105\,(0.248)\left(\frac{m_{\tilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [\epsilon_L(d)] \tag{6.85}$$
$$|\lambda'_{21k}| < 0.196\,(0.245)\left(\frac{m_{\tilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [\epsilon_L(d)] \tag{6.86}$$
$$|\lambda_{12k}| < 0.557\,(0.768)\left(\frac{m_{\tilde{\ell}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 1\sigma\,(2\sigma)\quad [\epsilon_R(d)] \tag{6.87}$$
$$|\lambda'_{2j1}| < 0.138\,(0.202)\left(\frac{m_{\tilde{d}_{Lj}}}{100\,\mathrm{GeV}}\right). \qquad 1\sigma\,(2\sigma)\quad [\epsilon_R(d)] \tag{6.88}$$

6.6 Forward-backward asymmetry in e^+e^- collisions

Another sensitive tests of the Standard model are the fermion pair production processes, $e^+e^- \to f\overline{f}$, where $f = \ell, q$. Mediated by photon or the Z boson in the SM, the asymmetric angular distribution of the produced particles is observed due to weak interactions, since the Z boson couplings differ for the left- and right-handed fermions. The forward-backward asymmetry A_{FB}^f in e^+e^- collisions, defined in eq. (6.89) below, can provide a measure of the

NC axial-vector couplings g_A^f. [12]

$$A_{FB}^f = \frac{F-B}{F+B} \equiv \frac{\displaystyle\int_0^1 \frac{d\sigma}{dcos\theta} dcos\theta - \int_{-1}^0 \frac{d\sigma}{dcos\theta} dcos\theta}{\displaystyle\int_{-1}^1 \frac{d\sigma}{dcos\theta} dcos\theta}. \tag{6.89}$$

To see how A_{FB}^f is related to the axial vector couplings let's consider the cross-section $\sigma(ff)$ for the $e^+e^- \to f\overline{f}$ process that can be generally written as: $\sigma(ff) = \sigma(QED) + \sigma(int) + \sigma(weak)$ [12] ; The middle term represents the interference between QED and the weak interactions. At the lowest order (Born approximation) one gets [12]:

$$\frac{d\sigma(ff)}{dcos\theta} = \frac{\pi\alpha^2}{2s}[R_{ff}(1+cos^2\theta) + B_{ff} cos\theta], \tag{6.90}$$

where θ is the center-of-mass scattering angle and [12]

$$R_{ff} = \frac{\sigma(ff)}{\sigma(QED)} = 1 - 2g_V^e g_V^f \chi + [(g_V^e)^2 + (g_A^e)^2][(g_V^f)^2 + (g_A^f)^2]\chi^2, \tag{6.91}$$

$$B_{ff} = -4g_A^e g_A^f \chi + 8(g_V^e g_V^f)(g_A^e g_A^f)\chi^2. \tag{6.92}$$

The B_{ff} term causes F-B asymmetry as can be observed in eq. (6.90). $\sigma(QED) = \frac{4\pi\alpha^2}{3s}$ and $\chi \equiv \frac{1}{4sin^2\theta_W cos^2\theta_W}\left(\frac{s}{M_Z^2 - s}\right)$ [12] in equations (6.91) and (6.92). The imaginary part of the propagator is small and thus is neglected. These results yield for F and B the following:

$$F = \int_0^1 \frac{d\sigma}{dcos\theta} dcos\theta = R_{ff} + \frac{1}{3}R_{ff} + \frac{1}{2}B_{ff}, \tag{6.93}$$

$$B = \int_{-1}^0 \frac{d\sigma}{dcos\theta} dcos\theta = R_{ff} + \frac{1}{3}R_{ff} - \frac{1}{2}B_{ff}. \tag{6.94}$$

$\Longrightarrow\ A_{FB}^f = \frac{F-B}{F+B} = \frac{3B_{ff}}{8R_{ff}}$. At sufficiently low energies, that is when $s \ll M_Z^2$, χ is a small number and, consequently, the χ^2 terms can be neglected in R_{ff} and B_{ff}:

$$\frac{B_{ff}}{R_{ff}} \approx \frac{-4g_A^e g_A^f \chi}{1 - 2g_V^e g_V^f \chi} \approx -4g_A^e g_A^f \chi(1 + 2g_V^e g_V^f \chi) = -4g_A^e g_A^f \chi + \mathcal{O}(\chi^2), \tag{6.95}$$

$$\Longrightarrow A_{FB}^f = -\frac{3}{2} g_A^e g_A^f \chi. \tag{6.96}$$

At the Z peak [3]

$$A_{FB}^{0,f} = \frac{3}{4} A^e A^f, \tag{6.97}$$

[12] Sometimes c_A^f is used instead of g_A^f.

where

$$A^f = \frac{2 g_V^f g_A^f}{g_V^{f2} + g_A^{f2}} \overset{\text{tree level}}{=} -T_3^f . \tag{6.98}$$

The presence of the $\not{R}_p$ interactions affects the forward-backward asymmetries A_{FB}^f by modifying axial coupling products through the t-channel diagrams with a sneutrino or squark exchange as shown in Fig. 14.

In order to get the corrections to $A^e A^f$ let's look directly at eq. (3) of Ref. [4] and take $f = \mu$. The calculations for other fermions will be similar. The relevant part comes from the third term of the above-mentioned equation [4]

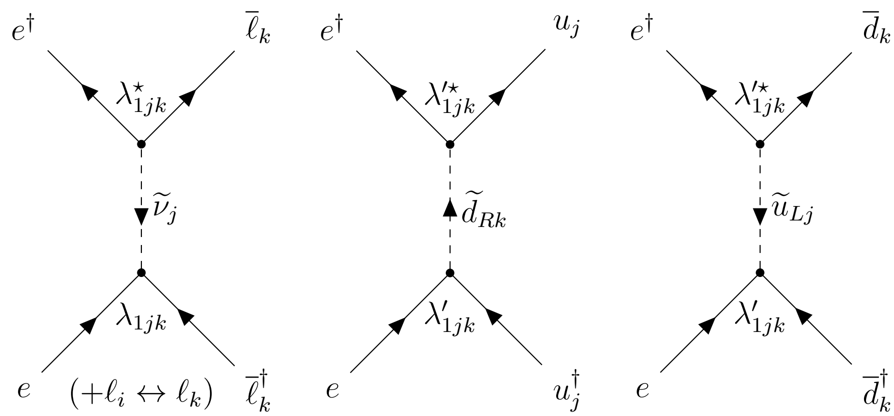


Figure 14: $\not{R}_p$ contributions to A_{FB}^f.

$$\begin{aligned} -\frac{1}{2}\frac{|\lambda_{1j2}|^2}{m^2_{\widetilde{\nu}_j}} [\overline{e}\gamma^\mu P_L e] [\overline{\mu}\gamma_\mu P_R \mu] &= -\frac{1}{8}\frac{|\lambda_{1j2}|^2}{m^2_{\widetilde{\nu}_j}} [\overline{e}\gamma^\mu e - \overline{e}\gamma^\mu\gamma_5 e] [\overline{\mu}\gamma_\mu \mu + \overline{\mu}\gamma_\mu\gamma_5\mu] \\ &= \frac{1}{8}\frac{|\lambda_{1j2}|^2}{m^2_{\widetilde{\nu}_j}} (\overline{e}\gamma^\mu\gamma_5 e)(\overline{\mu}\gamma_\mu\gamma_5\mu) + \ldots , \end{aligned}$$

where in the last step only the product of the axial-vector parts is specified as it is exactly the term that modifies A_{FB}. The contribution to A_{FB} in the SM at the tree-level is the following:

$$-\frac{1}{4}\frac{g^2}{M_W^2} [\overline{e}\gamma^\mu (g_V^e - g_A^e\gamma_5) e] [\overline{\mu}\gamma^\mu (g_V^\mu - g_A^\mu\gamma_5)\mu] = -\frac{1}{4}\frac{g^2}{M_W^2} [g_A^e g_A^\mu] (\overline{e}\gamma^\mu\gamma_5 e)(\overline{\mu}\gamma_\mu\gamma_5\mu) + \ldots ;$$

Adding these two contributions yields the modification of the SM value of the product of axial-vector couplings:

$$(g_A^e g_A^\mu)^{SM} \to (g_A^e g_A^\mu)^{SM} - \frac{1}{2}\left(\frac{M_W^2}{g^2}\right)\frac{|\lambda_{1j2}|^2}{m^2_{\widetilde{\nu}_j}} = \frac{1}{4} - \frac{1}{2} r_{1j2}(\widetilde{\nu}_j). \tag{6.99}$$

The $\not{R}_p$ contributions to other fermion asymmetries are calculated a similar way. I list the results of my calculations here:

$$A^e A^e = \frac{1}{4} - \frac{1}{2} r_{ijk}(\widetilde{\nu}_j), \qquad ijk = 121, 131, \tag{6.100}$$

$$A^e A^\mu = \frac{1}{4} - \frac{1}{2} r_{ijk}(\widetilde{\nu}_j), \qquad ijk = 122, 132, 211, 231, \tag{6.101}$$

$$A^e A^\tau = \frac{1}{4} - \frac{1}{2} r_{ijk}(\widetilde{\nu}_j), \qquad ijk = 123, 133, 311, 321, \tag{6.102}$$

$$A^e A^c = -\frac{1}{4} - \frac{1}{2} r'_{12k}(\widetilde{d}_{Rk}), \tag{6.103}$$

$$A^e A^s = \frac{1}{4} - \frac{1}{2} r'_{1j2}(\widetilde{u}_{Lj}), \tag{6.104}$$

$$A^e A^b = \frac{1}{4} - \frac{1}{2} r'_{1j3}(\widetilde{u}_{Lj}). \tag{6.105}$$

So for the Asymmetries at the Z peak eq. (6.97) yields the following expression:

$$\frac{A_{FB}^{0,f}}{(A_{FB}^{0,f})^{\rm SM}} = |1 + r^{(\prime)}|^{-2}. \tag{6.106}$$

Experimental and SM values (including radiative corrections) for $A_{FB}^{0,f}$ are [6]

$$A_{FB}^{0,e} = 0.0145 \pm 0.0025, \qquad (A_{FB}^{0,e})^{\rm SM} = 0.01619 \pm 0.00007, \tag{6.107}$$

$$A_{FB}^{0,\mu} = 0.0169 \pm 0.0013, \qquad (A_{FB}^{0,\mu})^{\rm SM} = 0.01619 \pm 0.00007, \tag{6.108}$$

$$A_{FB}^{0,\tau} = 0.0188 \pm 0.0017, \qquad (A_{FB}^{0,\tau})^{\rm SM} = 0.01619 \pm 0.00007, \tag{6.109}$$

$$A_{FB}^{0,c} = 0.0707 \pm 0.0035, \qquad (A_{FB}^{0,c})^{\rm SM} = 0.0736 \pm 0.0002, \tag{6.110}$$

$$A_{FB}^{0,s} = 0.0976 \pm 0.0114, \qquad (A_{FB}^{0,s})^{\rm SM} = 0.1031 \pm 0.0002, \tag{6.111}$$

$$A_{FB}^{0,b} = 0.0996 \pm 0.0016, \qquad (A_{FB}^{0,b})^{\rm SM} = 0.1030 \pm 0.0002. \tag{6.112}$$

After plugging eq.s (6.107) - (6.112) into eq. (6.106), I obtain the following single bounds:

$$|\lambda_{ijk}| < 0.287\,(0.362) \left(\frac{m_{\widetilde{\nu}_j}}{100\,\text{GeV}}\right), \qquad ijk = 121, 131,\ \ 1\sigma\,(2\sigma) \quad \left[A_{FB}^{0,e}\right] \tag{6.113}$$

$$|\lambda_{ijk}| < 0.108\,(0.193) \left(\frac{m_{\widetilde{\nu}_j}}{100\,\text{GeV}}\right), \qquad ijk = 122, 132, 211, 231,\ \ 1\sigma\,(2\sigma) \quad \left[A_{FB}^{0,\mu}\right] \tag{6.114}$$

$$|\lambda_{ijk}| < 0.125 \left(\frac{m_{\widetilde{\nu}_j}}{100\,\text{GeV}}\right), \qquad ijk = 123, 133, 311, 321,\ \ 2\sigma\,(1\sigma\,\text{excluded}) \quad \left[A_{FB}^{0,\tau}\right] \tag{6.115}$$

$$|\lambda'_{12k}| < 0.166\,(0.207) \left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\text{GeV}}\right), \qquad 1\sigma\,(2\sigma) \quad \left[A_{FB}^{0,c}\right] \tag{6.116}$$

$$|\lambda'_{1j2}| < 0.288\,(0.296) \left(\frac{m_{\widetilde{u}_{Lj}}}{100\,\text{GeV}}\right), \qquad 1\sigma\,(2\sigma) \quad \left[A_{FB}^{0,s}\right] \tag{6.117}$$

$$|\lambda'_{1j3}| = 0.102 \begin{array}{l} +0.022\,(+0.041) \\ -0.028\,(-0.078) \end{array} \left(\frac{m_{\widetilde{u}_{Lj}}}{100\,\text{GeV}}\right). \qquad 1\sigma\,(2\sigma) \quad \left[A_{FB}^{0,b}\right] \tag{6.118}$$

6.7 Atomic parity violation

Electroweak theory violates parity as it treats left- and right-handed objects differently. Precise measurements of parity violation in a number of different atoms provide important tests of the SM at low energies. Additionally, comparing a measured value of atomic parity violation (APV) with the corresponding theoretical value predicted by the SM can also provide probes of physics beyond the SM, since APV can be sensitive to new physics with parity violating interactions. The SM prediction requires, as input, the mass of the Z boson and the electronic structure of the atom in question [13]. Even though M_Z is measured with a very high precision, the uncertainties in the atomic structure can still be significant. Therefore atoms with accurately known structure, one of them being caesium, are used in variety of precision experiments. The structure of caesium is well-known because it is an alkali atom with a single valence electron outside of a tightly bound inner core [13]. APV has been observed via the $6S \to 7S$ transitions of $^{133}_{55}\mathrm{Cs}$ [13].

Mediated by the Z-boson exchange between the atomic electrons and the nucleus in the SM, APV transitions measure the parity-violating couplings in the electron-hadron interactions that can be represented by the following four-fermion effective Lagrangian [3]:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \sum_{q=u,d} \left[C_1^{SM}(q)(\bar{e}\gamma^\mu\gamma_5 e)(\bar{u}\gamma_\mu u) + C_2^{SM}(q)(\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu\gamma_5 u) \right], \tag{6.119}$$

where the parity-violating C couplings are defined at the tree level as [3]

$$\begin{aligned} .C_1^{SM}(u) &\equiv 2g_A^e g_V^u = -\frac{1}{2} + \frac{4}{3}s_W^2 \ , \ C_1^{SM}(d) \equiv 2g_A^e g_V^d = \frac{1}{2} - \frac{2}{3}s_W^2 \\ C_2^{SM}(u) &\equiv 2g_V^e g_A^u = -\frac{1}{2} + 2s_W^2 \ , \ C_2^{SM}(d) \equiv 2g_V^e g_A^d = \frac{1}{2} - 2s_W^2. \end{aligned} \tag{6.120}$$

In the presence of the $\not{R}_p$ couplings λ'_{11k} (λ'_{1j1}) additional parity-violating interactions arise that modify C coefficients through the s-channel (t-channel) exchange of $\widetilde{d}_{Rk}$ ($\widetilde{u}_{Lj}$) between an electron and a u (d) quark in the atomic nucleus as shown in Fig. 15. The simplest way to get the modified expressions for $C_1(q)$ and $C_2(q)$ is to add the relevant terms of the SM and $\not{R}_p$ Lagrangians. For $q = u$, the $\not{R}_p$ four-fermion interactions come from the first term of eq. (3) of Ref. [4] after replacing $\lambda \to \lambda'$, $\nu_L \to u_L$ and $e_R \to d_R$:

$$\begin{aligned} .\frac{|\lambda'_{11k}|^2}{2m^2_{\tilde{d}_{Rk}}}[\bar{e}\gamma^\mu P_L e][\bar{u}\gamma_\mu P_L u] &= \frac{|\lambda'_{11k}|^2}{8m^2_{\tilde{d}_{Rk}}}[\bar{e}\gamma^\mu e - \bar{e}\gamma^\mu\gamma_5 e][\bar{u}\gamma_\mu u - \bar{u}\gamma_\mu\gamma_5 u] \\ &= \frac{|\lambda'_{11k}|^2}{8m^2_{\tilde{d}_{Rk}}}[-(\bar{e}\gamma^\mu\gamma_5 e)(\bar{u}\gamma_\mu u) - (\bar{e}\gamma^\mu e)(\bar{u}\gamma_\mu\gamma_5 u) + \cdots], \end{aligned} \tag{6.121}$$

where in the last step only the $g_A g_V$ terms are written out explicitly.

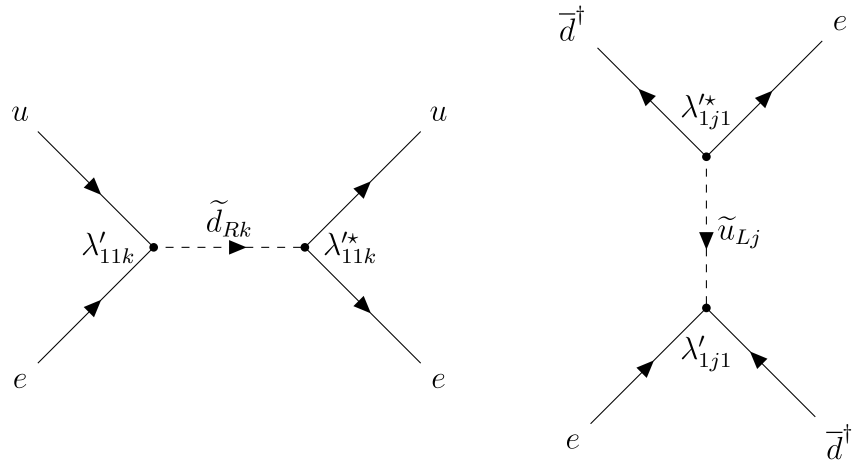
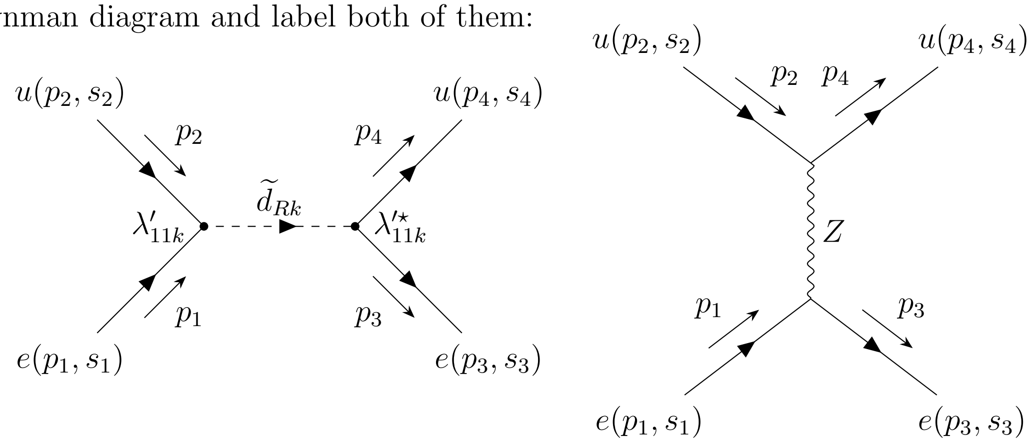


Figure 15: $\not{R}_p$ contributions to atomic parity violation.

Adding eq. (6.121) and eq. (6.119), while also keeping in mind that G_F is itself modified by λ_{12k}, yields for $C_{1/2}(u)$ the following expression:

$$C_{1/2}(u) = C_{1/2}^{SM}(u)[1 - r_{12k}(\widetilde{e}_{Rk})] - \frac{M_W^2}{g^2}\frac{|\lambda'_{11k}|^2}{m^2_{\widetilde{d}_{Rk}}} = C_{1/2}^{SM}(u)[1 - r_{12k}(\widetilde{e}_{Rk})] - r'_{11k}(\widetilde{d}_{Rk}). \quad (6.122)$$

One could also arrive to the above expression by directly calculating the matrix elements of Fig 15. The first diagram of Fig. 15 modifies the $g_L g_L$ coupling and, consequently, the $g_A g_V$ coupling since $g_A g_V = (g_L - g_R)(g_L + g_R) = g_L g_L + \cdots$. Let's also include the relevant SM Feynman diagram and label both of them:



These diagrams at low energies yield the following matrix elements:

$$\begin{cases} \mathcal{M}^{\not{R}_p} &= \dfrac{|\lambda'_{11k}|^2}{m^2_{\widetilde{d}_{Rk}}}\{x_1 x_2\}\{x_3^\dagger x_4^\dagger\} \overset{(2.18)}{=} -\dfrac{|\lambda'_{11k}|^2}{2m^2_{\widetilde{d}_{Rk}}}\{x_1\sigma^\mu x_3^\dagger\}\{x_2\sigma_\mu x_4^\dagger\}. \\ \mathcal{M}^{SM} &= -\dfrac{g^2}{M_W^2}(g_L^e g_L^u)^{SM}\{x_1\sigma^\mu x_3^\dagger\}\{x_2\sigma_\mu x_4^\dagger\}. \end{cases} \quad (6.123)$$

There is a relative minus sign between the two matrix elements due to the ordering of the external fermions so that $g^e_L g^u_L = (g^e_L g^u_L)^{SM} - \frac{1}{2} r'_{11k}(\widetilde{d}_{Rk})$. This, combined with the fact that G_F is itself modified by the λ_{12k} coupling, gives the same expression for $C_{1/2}(u)$.

The relevant $\not{R}_p$ four-fermion interactions for the d quarks come from the third term of eq. (3) of Ref. [4] after replacing $\lambda \to \lambda'$, $\nu_L \to u_L$ and $e_R \to d_R$:

$$-\frac{|\lambda'_{1j1}|^2}{2m^2_{\widetilde{u}_{Lk}}}[\bar{e}\gamma^\mu P_L e][\bar{d}\gamma_\mu P_R d] = -\frac{|\lambda'_{1j1}|^2}{8m^2_{\widetilde{u}_{Lj}}}[-(\bar{e}\gamma^\mu\gamma_5 e)(\bar{d}\gamma_\mu d) + (\bar{e}\gamma^\mu e)(\bar{d}\gamma_\mu\gamma_5 d) + \cdots]. \quad (6.124)$$

So the in the presence of $\not{R}_p$ interactions $C_1(d)$ and $C_2(d)$ read:

$$C_1(d) = (\frac{1}{2} - \frac{2}{3}s^2_W)[1 - r_{12k}(\widetilde{e}_{Rk})] + r'_{1j1}(\widetilde{u}_{Lj}), \quad (6.125)$$

$$C_2(d) = (\frac{1}{2} - 2s^2_W)[1 - r_{12k}(\widetilde{e}_{Rk})] - r'_{1j1}(\widetilde{u}_{Lj}). \quad (6.126)$$

Another way to impose bounds on the coupling constants involved is to use the weak charge Q_W, which plays the same role for the Z exchange as the electric charge does for the Coulomb interaction. The weak charge is defined in terms of C coefficients as [3]

$$Q_W := -2\left[(A+Z)C_1(u) + (2A-Z)C_1(d)\right]. \quad (6.127)$$

Here Z is the atomic number and A is the nucleon number. Q_W is determined by an electric dipole transition amplitude $E_{PV} = kQ_W$ between two atomic states with the same parity, such as the $6S$ and $7S$ states in caesium with k being an atomic-structure factor [14]. One could evaluate bounds on the couplings directly from the expressions for $C_{1/2}(u/d)$, but the bounds obtained from the weak charge calculations are more stringent. The quantity of interest here is actually the difference between the measured weak charge and the SM value, $\delta Q_W = Q_W - Q^{SM}_W$ [3]. Below I calculate δQ_W for a stable isotope of caesium, $^{133}_{55}$Cs:

$$\begin{aligned} Q_W(\mathrm{Cs}) =& -2[\,(A+Z)^{Cs}\{C^{SM}_1(u) - C^{SM}_1(u)r_{12k}(\widetilde{e}_{Rk}) - r'_{11k}(\widetilde{d}_{Rk})\} \\ &+ (2A-Z)^{Cs}\{C^{SM}_1(d) - C^{SM}_1(d)r_{12k}(\widetilde{e}_{Rk}) + r'_{1j1}(\widetilde{u}_{Lj})\}\,], \quad (6.128) \\ Q^{SM}_W(\mathrm{Cs}) =& -2[(A+Z)^{Cs}C^{SM}_1(u) + (2A-Z)^{Cs}C^{SM}_1(d)], \quad (6.129) \\ \Longrightarrow \delta Q_W(\mathrm{Cs}) =& -2[-(A+Z)^{Cs}r'_{11k}(\widetilde{d}_{Rk}) + (2A-Z)^{Cs}r'_{1j1}(\widetilde{u}_{Lj}) \\ &- \{(A+Z)^{Cs}C^{SM}_1(u) + (2A+Z)^{Cs}C^{SM}_1(d)\}r_{12k}(\widetilde{e}_{Rk})] \\ =& \, 376\, r'_{11k}(\widetilde{d}_{Rk}) - 422\, r'_{1j1}(\widetilde{u}_{Lj}) - Q^{SM}_W r_{12k}(\widetilde{e}_{Rk}). \quad (6.130) \end{aligned}$$

Now I can finally set bounds on the coupling constants involved. The SM and measured values are $Q^{\mathrm{SM}}_W(^{133}_{55}\mathrm{Cs}) = -73.23 \pm 0.01$ and $Q_W(^{133}_{55}\mathrm{Cs}) = -72.82 \pm 0.42$ respectively [15]. These numbers together with eq. (6.130) give me the following single bounds:

$$|\lambda'_{11k}| < 0.046\left(\frac{m_{\widetilde{d}_{Rk}}}{100\,\mathrm{GeV}}\right), \qquad 2\sigma \quad (6.131)$$

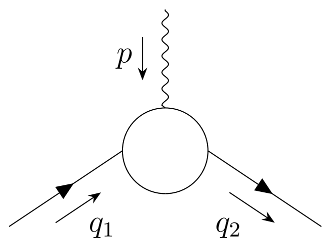
$$|\lambda'_{1j1}| < 0.025 \left(\frac{m_{\tilde{u}_{Lj}}}{100\,\mathrm{GeV}}\right), \qquad 2\sigma \tag{6.132}$$

$$|\lambda_{12k}| < 0.104 \left(\frac{m_{\tilde{e}_{Rk}}}{100\,\mathrm{GeV}}\right). \qquad 2\sigma \tag{6.133}$$

6.8 The anomalous magnetic moment of muon

The last low energy process that I discuss concerns the anomalous magnetic dipole moment of muon. The g-factor of muon represents the relative strength of its intrinsic magnetic dipole moment to the strength of the spin-orbit coupling and in Dirac theory $g = 2$ [5]. Any difference from $g = 2$ is dubbed as particle having the anomalous magnetic dipole moment.

Corrections to the magnetic moment come from diagrams that modify the way photons interact with spinors. A generic diagram is shown below [5]:



where $p^\mu = q_2^\mu - q_1^\mu$. Associated matrix element can be parametrized in an usual four-component notation in the following way [5]:

$$i\mathcal{M}^\mu = (-ie)\,\overline{u}(q_2)\left[F_1\left(\frac{p^2}{m^2}\right)\gamma^\mu + \frac{i\sigma^{\mu\nu}}{2m}p_\nu F_2\left(\frac{p^2}{m^2}\right)\right]u(q_1), \tag{6.134}$$

where F_1 and F_2 are independent form factors. The tree-level graph, for example, corresponds to $F_1 = 1$ and $F_2 = 0$.

Form factor F_1 simply modifies coupling e of the interaction $A_\mu\overline{\psi}\gamma^\mu\psi$, giving it scale dependence [5]. This means, that F_1 term is not relevant for anomalous magnetic moment discussions. On the other hand, the second term in (6.134) has exactly the structure we are looking for. Relativistically the value of $g = 2$ comes from $\sigma^{\mu\nu}$ term of the Dirac equation coupled to an external magnetic field. Without F_2 $g = 2$, so it follows that this form factor modifies the magnetic moment at the scale p^2 by $g \to 2 + 2F_2(p^2/m^2)$ [5]. Since the measurements are performed at non-relativistic energies, with a good approximation we have that [5]

$$g = 2 + 2F_2(0). \tag{6.135}$$

In this way, the whole problem of finding radiative corrections to g reduces to calculating $F_2(0)$, which in the literature is often denoted by a_μ.

The loop correction to the magnetic moment of μ arises if there is a chirality flip between the external fermions as eq. (6.134) suggests. Indeed we have:

$$\overline{u}\sigma^{\mu\nu}u = \overline{(u_L + u_R)}\sigma^{\mu\nu}(u_L + u_R) = (\overline{u}_L + \overline{u}_R)\sigma^{\mu\nu}(u_L + u_R).$$

Let's focus on the $\overline{u}_L u_L$ term (or it could be the $\overline{u}_R u_R$ term) and rewrite it in terms of projection operators:

$$\overline{u}_L\sigma^{\mu\nu}u_L = \overline{(P_L u)}\sigma^{\mu\nu}P_L u = \overline{u}P_R\sigma^{\mu\nu}P_L u.$$

Now we can move P_R through $\sigma^{\mu\nu} \propto [\gamma^\mu, \gamma^\nu]$. Keeping in mind that $\{\gamma_5, \gamma^\mu\} = 0$ we get that

$$\begin{aligned}\overline{u}P_R\sigma^{\mu\nu}P_L &\propto \overline{u}P_R[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]P_L u = \overline{u}[\gamma^\mu P_L\gamma^\nu - \gamma^\nu P_L\gamma^\mu]P_L u \\ &= \overline{u}[\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu]P_R P_L u = 0.\end{aligned}$$

Thus only the mixed terms survive. This chirality transition is usually done by an insertion of a fermion mass and, consequently, a magnetic moment is then proportional to the external fermion mass [16]. Since in the SM there is no heavy fermion one does not expect an anomalously large anomalous magnetic moment of μ. Actually, the measured value of the anomalous magnetic moment of muon is of order predicted in the SM, off from the theoretical prediction by only a few sigma levels. Even though the existence of new physics contributing to a_μ can not be definitely concluded, still one can search for possibilities of generating a_μ of order the electroweak scale [16]. Since the experimental value is of order the expected theoretical prediction, the mass scale of new physics must be close to the electroweak scale.

One particular model in which this can be achieved was discussed by Kim et al., Ref. [16]. They studied the anom. magn. moment of μ in the so-called effective supersymmetric theory (ESUSY [17]) . In the ESUSY, sparticle masses of the first two generations are of order 20 TeV [17], so they decouple. This means that the ESUSY with R-parity conservation cannot account for the possible extra contributions to a_μ, since R-parity conserving loops involve $\widetilde{\nu}_\mu$ and $\widetilde{\mu}$ that are too heavy in the ESUSY [17].

The situation is different for the third generation sparticles, which in the ESUSY can be taken to be lighter than 1 TeV [17]. So, following Ref. [16], I will discuss the anomalous magnetic moment of muon in the ESUSY with R-parity violation involving contributions from the third generation sparticles only.

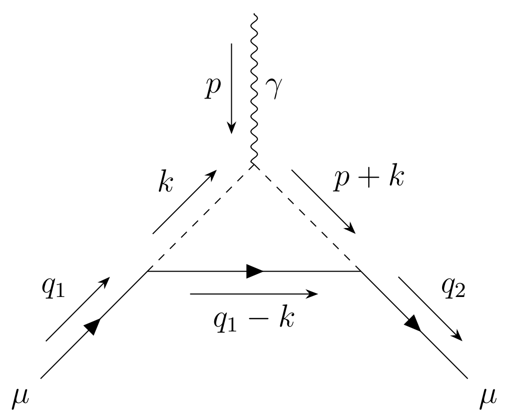
The required chirality transition can also take place on the internal fermion and sfermion lines because of the possible mixing between the left and right sfermions. However, such contributions can be neglected due to the chiral nature of the SM and the fact that neutrinos are very light [16].

The $\not{R}_p$ contributions to a_μ from the λ and λ' couplings are shown in Fig. 16 [16]. There are two types of loops involved: one with two scalars and one fermion, and another with two fermions and one scalar. In what follows I calculate both of these loop types separately in the four-component notation and obtain expressions for $F_2(0)$. The four-component version of Lagrangians (5.15) and (5.16) can be found in section 2.1 of Ref. [3]. Neglecting generation indices, fermionic part of the interactions are of the following form:

$$\overline{f}_R f_L = \overline{(P_R f)} P_L f = (P_R f)^\dagger \gamma_0 P_L f = f^\dagger P_R \gamma_0 P_L f = f^\dagger \gamma_0 P_L P_L f = \overline{f} P_L f,$$

while the h.c. interactions contain P_R instead of P_L in the last term as can be easily verified by doing similar calculation. Therefore, in four-component notation matrix elements will generally contain P_L and P_R.

First I calculate a loop with two scalars and one fermion. It is shown below with the momenta assignments. λ denotes a generic Yukawa coupling constant. From the momentum conservation it follows that $p = q_2 - q_1$.



$$i\mathcal{M}^\mu = \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \left[i\frac{\lambda}{2}(1-\gamma_5)\right] \left\{ \frac{i}{(p+k)^2 - m_s^2 + i\epsilon} \right\} \left[iQ_s e \left(2k^\mu + p^\mu\right)\right]$$
$$\times \left\{ \frac{i}{k^2 - m_s^2 + i\epsilon} \right\} \left\{ \frac{i\left(\not{q}_1 - \not{k} + m_f\right)}{(q_1-k)^2 - m_f^2 + i\epsilon} \right\} \left[i\frac{\lambda^*}{2}(1+\gamma_5)\right] u(q_1)$$
$$= \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{\left(-Q_s e \left|\lambda^2\right|/4\right)\left(2k^\mu + p^\mu\right)(1-\gamma_5)\left(\not{q}_1 - \not{k} + m_f\right)(1+\gamma_5)}{\left[(p+k)^2 - m_s^2 + i\epsilon\right]\left[k^2 - m_s^2 + i\epsilon\right]\left[(q_1-k)^2 - m_f^2 + i\epsilon\right]} u(q_1)$$
$$= \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{g\left(2k^\mu + p^\mu\right)\left\{\left(\not{q}_1 - \not{k}\right)(1+\gamma_5)(1+\gamma_5) + m_f(1-\gamma_5)(1+\gamma_5)\right\}}{\left[(p+k)^2 - m_s^2 + i\epsilon\right]\left[k^2 - m_s^2 + i\epsilon\right]\left[(q_1-k)^2 - m_f^2 + i\epsilon\right]} u(q_1)$$
$$= \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{g\left(2k^\mu + p^\mu\right)\left\{\left(\not{q}_1 - \not{k}\right)(2+2\gamma_5) + m_f \cancelto{o}{(1-1)}\right\}}{\left[(p+k)^2 - m_s^2 + i\epsilon\right]\left[k^2 - m_s^2 + i\epsilon\right]\left[(q_1-k)^2 - m_f^2 + i\epsilon\right]} u(q_1)$$

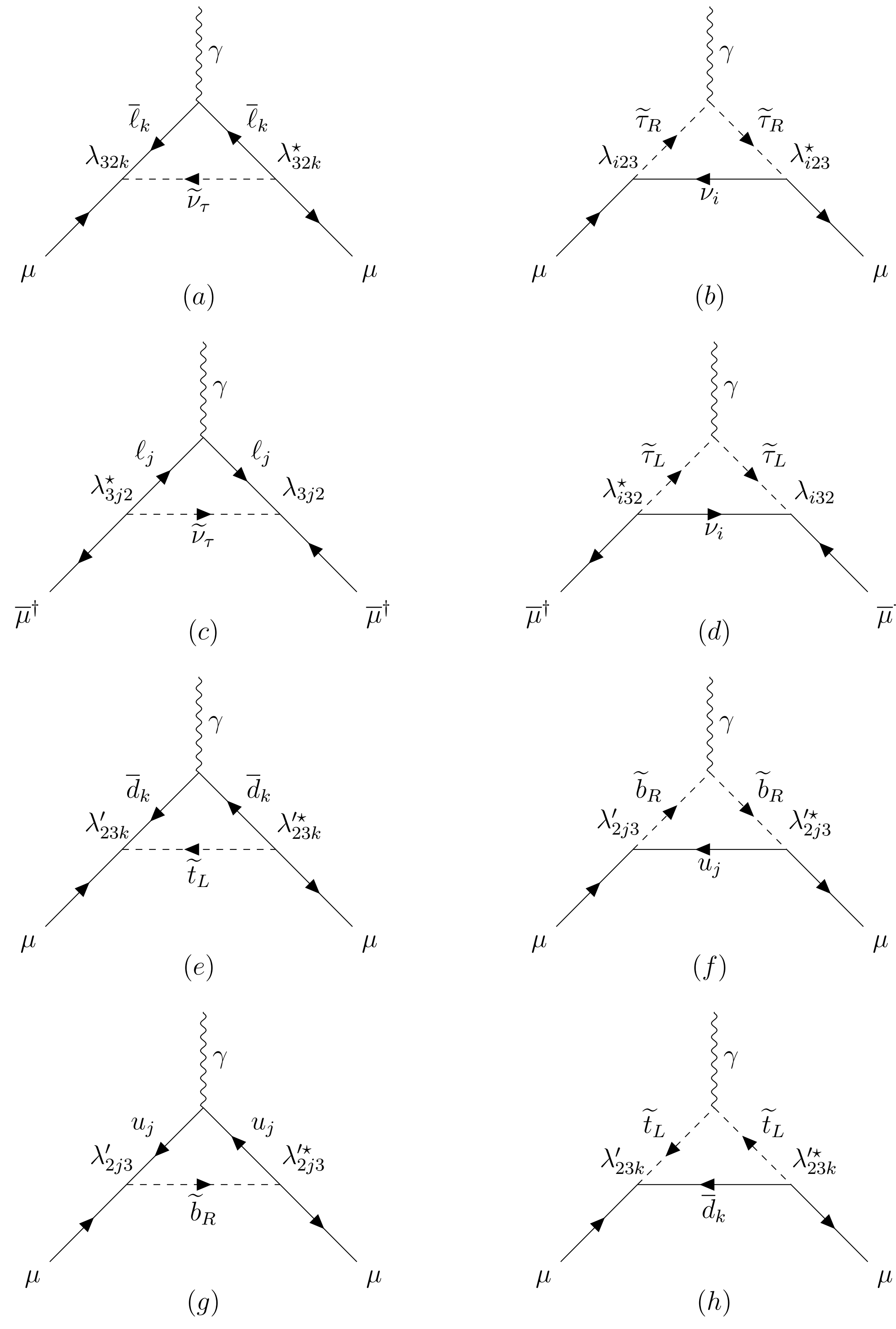


Figure 16: $\not{R}_p$ contributions to a_μ from the λ and λ' couplings. In each diagram one of the external muons is chirally flipped by m_μ insertion.

$$= 2g\,\overline{u(q_2)} \int\limits_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{(\not{q}_1 - \not{k})\,(2k^\mu + p^\mu)\,(1+\gamma_5)}{\left[(p+k)^2 - m_s^2 + i\epsilon\right]\left[k^2 - m_s^2 + i\epsilon\right]\left[(q_1-k)^2 - m_f^2 + i\epsilon\right]} u(q_1),$$

where $g \equiv (-Q_s e\,|\lambda^2|\,/4)$; To simplify the above expression let's first complete the square in the denominator by introducing the Feynman identity [5]

$$\frac{1}{ABC} = 2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{1}{[Ax+By+Cz]^3}. \tag{6.136}$$

In our case let $A \equiv k^2 - m_s^2 + i\epsilon$, $B \equiv (p+k)^2 - m_s^2 + i\epsilon$, $C \equiv (q_1-k)^2 - m_f^2 + i\epsilon$.

The denominator:

$$\begin{aligned} Ax + By + Cz =& \left(k^2 - m_s^2 + i\epsilon\right)x + \left((p+k)^2 - m_s^2 + i\epsilon\right)y + \left((q_1-k)^2 - m_f^2 + i\epsilon\right)z \\ =& k^2x - m_s^2x + p^2y + k^2y + 2pky - m_s^2y + q_1^2z + k^2z - 2q_1kz - m_f^2z + i\epsilon \\ =& k^2 + 2k(yp - zq_1) + yp^2 + zq_1^2 - m_s^2(x+y) - m_f^2z + i\epsilon. \end{aligned}$$

Complete the square: $(k^\mu + yp^\mu - zq_1^\mu)^2 = k^2 + 2k(yp - zq_1) + (yp^\mu - zq_1^\mu)^2$.

$$\begin{aligned} (yp^\mu - zq_1^\mu)^2 =& y^2p^2 + z^2q_1^2 - 2yzpq_1 = (1-x-z)yp^2 + (1-x-y)zq_1^2 - 2yzpq_1 \\ =& yp^2 - xyp^2 - yzp^2 + zq_1^2 - xzq_1^2 - yzq_1^2 - 2yzpq_1 \\ =& yp^2 + zq_1^2 - xyp^2 - yzp^2 - 2yzpq_1 - (xz + yz)q_1^2 \\ =& yp^2 + zq_1^2 - xyp^2 - yzp^2 - 2yzpq_1 - z(1-z)m_\mu^2 \\ =& yp^2 + zq_1^2 - xyp^2 - z(1-z)m_\mu^2 - yzp^2 - 2yzpq_1. \end{aligned}$$

Considering only the last 2 terms:

$$\begin{aligned} -yzp^2 - 2yzpq_1 =& -yz(q_2 - q_1)^2 - 2yz(q_2 - q_1)q_1 = -yz(q_2^2 + q_1^2 - 2q_2q_1) \\ & - 2yz(q_2q_1 - q_1^2) = -yzm_\mu^2 - yzm_\mu^2 + 2yzq_2q_1 - 2yzq_2q_1 + 2yzm_\mu^2 \\ =& 0. \end{aligned}$$

$$\Longrightarrow (yp^\mu - zq_1^\mu)^2 = yp^2 + zq_1^2 - xyp^2 - z(1-z)m_\mu^2.$$

From these results it follows that

$$\begin{aligned} Ax + By + Cz =& (k^\mu + yp^\mu - zq_1^\mu)^2 + xyp^2 + z(1-z)m_\mu^2 - (1-z)m_s^2 - zm_f^2 + i\epsilon \\ =& (k^\mu + yp^\mu - zq_1^\mu)^2 - \Delta + i\epsilon, \end{aligned}$$

where $\Delta \equiv -xyp^2 - z(1-z)m_\mu^2 + (1-z)m_s^2 + zm_f^2$. So, if one shifts $k^\mu \to k^\mu - yp^\mu + zq_1^\mu$, the denominator becomes $\boxed{(k^2 - \Delta + i\epsilon)^3.}$

The numerator

$N^\mu = \overline{u(q_2)}\,(\not q_1 - \not k)\,(2k^\mu + p^\mu)\,(1+\gamma_5)\,u(q_1).$

One can use momentum space Dirac equations and the Gordon identity to simplify the above expression:

$$\begin{cases} \overline{u(q_2)}\,\not q_2 &= m_\mu \overline{u(q_2)} \\ \not q_1 u(q_1) &= m_\mu u(q_1) \end{cases} ; \quad \overline{u(q_2)}(q_2^\mu + q_1^\mu)u(q_1) = 2m_\mu \overline{u(q_2)}\gamma^\mu u(q_1) - i\overline{u(q_2)}\sigma^{\mu\nu}p_\nu u(q_1).$$

After shifting k^μ the first two brackets of N^μ become: [13]

$$\begin{aligned}
&2\not q_1\,(k^\mu - yp^\mu + zq_1^\mu\,) - 2\,(\not k - y\not p + z\not q_1)\,(k^\mu - yp^\mu + zq_1^\mu) + \not q_1 p^\mu - (\not k - y\not p + z\not q_1)\,p^\mu \\
&= 2\not q_1 k^\mu - 2y\,\not q_1 p^\mu + 2z\,\not q_1 q_1^\mu - 2(\not k k^\mu - y\,\not p k^\mu + z\,\not q_1 k^\mu - y\,\not k p^\mu + y^2\,\not p p^\mu - yz\,\not q_1 p^\mu \\
&\quad + z\not k q_1^\mu - yz\,\not p q_1^\mu + z^2\,\not q_1 q_1^\mu) + \not q_1 p^\mu - \not k p^\mu + y\,\not p p^\mu - z\not q_1 p^\mu \\
&= (2-2z)\cancelto{o}{k^\mu \not q_1} + (1-2y)p^\mu \not q_1 + (2z-2z^2)q_1^\mu \not q_1 - 2k^\mu \not k + 2y\,\cancelto{o}{k^\mu \not p} + (2y-1)\cancelto{o}{p^\mu \not k} \\
&\quad + (y-2y^2)p^\mu \not p + (2yz - z)p^\mu \not q_1 - 2z\,\cancelto{o}{q_1^\mu \not k} + 2yz\,q_1^\mu \not p \\
&= (1-2y+2yz-z)p^\mu \not q_1 + 2z(1-z)q_1^\mu \not q_1 - \frac{1}{2}g^{\mu\nu}k^2\gamma_\nu + y(1-2y)p^\mu \not p + 2yz\,q_1^\mu \not p.
\end{aligned}$$

Neglecting k^2 term as it is not relevant for the anomalous magnetic moment calculation we are left with $(1-2y+2yz-z)p^\mu \not q_1 + 2z(1-z)q_1^\mu \not q_1 + y(1-2y)p^\mu \not p + 2yz\,q_1^\mu \not p$. Simplifying the factors using the fact that $x+y+z=1$ gives [14]

$$\begin{cases} 1 - z - 2y + 2yz = x + y - 2y + 2yz = \cancelto{o}{(x-y)} + 2yz, \\ y(1-2y) = y(x+y+z-2y) = y\,\cancelto{o}{(x-y)} + yz. \end{cases}$$

So one is left with the following expression: $\{2yz\,p^\mu \not q_1 + 2z(1-z)q_1^\mu \not q_1 + yz\,p^\mu \not p + 2yz\,q_1^\mu \not p\}$.

$$\begin{aligned}
\Longrightarrow\quad N^\mu &= \{...\}\,(1+\gamma_5) = 2yz\,p^\mu m_\mu(1-\gamma_5) + 2z(1-z)q_1^\mu m_\mu(1-\gamma_5) \\
&\quad + \ (yz\,p^\mu \not q_2 - yz\,p^\mu \not q_1 + 2yz\,q_1^\mu \not q_2 - 2yz\,q_1^\mu \not q_1)(1+\gamma_5) \\
&= 2yz\,p^\mu m_\mu(1-\gamma_5) + 2z(x+y)q_1^\mu m_\mu(1-\gamma_5) + yz\,p^\mu m_\mu(1+\gamma_5) \\
&\quad - yz\,p^\mu m_\mu(1-\gamma_5) + 2yz\,q_1^\mu m_\mu(1+\gamma_5) - 2yz\,q_1^\mu m_\mu(1-\gamma_5) \\
&= yz\,p^\mu m_\mu(1-\gamma_5) + \underline{2xz\,q_1^\mu m_\mu(1-\gamma_5)} + \underline{2yz\,q_1^\mu m_\mu(1-\gamma_5)} + yz\,p^\mu m_\mu(1+\gamma_5)
\end{aligned}$$

[13] For simplicity I will not carry around u spinors, but naturally the following expressions are implied to be sandwiched between $\overline{u(q_2)}$ and $\overline{u(q_1)}$

[14] $(x-y)$ parts will drop after the integration since the integrand and Δ are both symmetric under $x \leftrightarrow y$.

$$
\begin{aligned}
&+\ 2yz\, q_1^\mu m_\mu(1+\gamma_5) - \underline{2yz\, q_1^\mu m_\mu(1-\gamma_5)} \\
&= yz\, p^\mu m_\mu(2) + 2yz\, q_1^\mu m_\mu(1-\gamma_5) + 2yz\, q_1^\mu m_\mu(1+\gamma_5) \\
&= 2yz\, m_\mu p^\mu + 2yz\, m_\mu q_1^\mu(2) \\
&= 2yz\, m_\mu q_2^\mu - 2yz\, m_\mu q_1^\mu + 4yz\, m_\mu q_1^\mu \\
&= 2yz\, m_\mu q_2^\mu + 2yz\, m_\mu q_1^\mu .
\end{aligned}
$$

Therefore $N^\mu = 2yz\, m_\mu(q_2^\mu + q_1^\mu) = 2yz\, m_\mu \left[2m_\mu\gamma^\mu - \underline{i\sigma^{\mu\nu}p_\nu}\right]$. The $\sigma^{\mu\nu}p_\nu$ part of N^μ is exactly what we are looking for. Thus, recalling that F_2 was defined as the coefficient of this operator normalized by $\frac{2m}{e}$, the contribution to $F_2(0)$ is

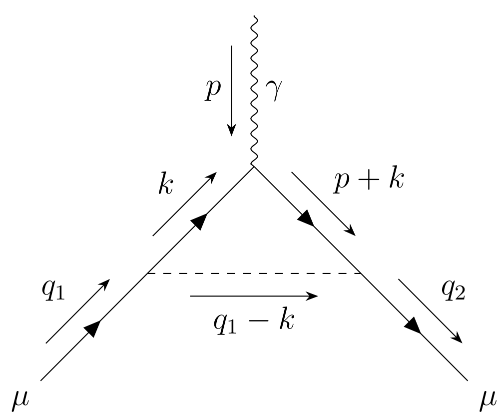
$$
\begin{aligned}
F_2(0) &= \frac{2m_\mu}{e}\left(-2Q_s e\frac{|\lambda|^2}{4}\right) 2\int_0^1 dx\, dy\, dz\, \delta(x+y+z-1) \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4}\frac{-2yzm_\mu i}{(k^2-\Delta+i\epsilon)^3} \\
&= \left(-Q_s|\lambda|^2\right) 4m_\mu^2\,(-1)\int_0^1 dx\, dy\, dz\, \delta(x+y+z-1)\, yzi\left[\frac{-i}{32\pi^2\Delta}\right] \\
&= \left(-Q_s|\lambda|^2\right)\frac{m_\mu^2}{8\pi^2}(-1)\int_0^1 dz\,\frac{z}{\Delta}\int_0^{1-z} dy\, y \\
&= \left(-Q_s|\lambda|^2\right)\frac{m_\mu^2}{8\pi^2}(-1)\int_0^1 dz\,\frac{z(1-z)^2}{2\Delta}.
\end{aligned}
$$

Change of variables: $z = 1-x \implies x = 1-z$ & $dz = -dx$. In terms of x $F_2(0)$ takes the following form:

$$
F_2(0) = -Q_s|\lambda|^2\frac{m_\mu^2}{16\pi^2}\int_0^1 dx\,\frac{x^3-x^2}{\Delta}, \tag{6.137}
$$

with $\Delta = m_\mu^2 x^2 + (m_s^2 - m_\mu^2)x + m_f^2(1-x)$.

Now that I have finally obtained eq. (6.137) let's proceed to computing the second type one-loop diagram (two fermions and one scalar). It is shown below with the similar momenta assignments.



$$i\mathcal{M}^\mu = \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \left[i\frac{\lambda}{2}(1-\gamma_5)\right] \left\{\frac{i(\not{p}+\not{k}+m_f)}{(p+k)^2 - m_f^2 + i\epsilon}\right\} [iQ_f e\gamma^\mu]$$

$$\times \left\{\frac{i(\not{k}+m_f)}{k^2 - m_f^2 + i\epsilon}\right\} \left[i\frac{\lambda^*}{2}(1+\gamma_5)\right] \left\{\frac{i}{(q_1-k)^2 - m_s^2 + i\epsilon}\right\} u(q_1)$$

$$= \overline{u(q_2)} \int_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{(-Q_f e\,|\lambda^2|/4)(1-\gamma_5)(\not{p}+\not{k}+m_f)\gamma^\mu(\not{k}+m_f)(1+\gamma_5)}{\left[(p+k)^2 - m_f^2 + i\epsilon\right]\left[k^2 - m_f^2 + i\epsilon\right]\left[(q_1-k)^2 - m_s^2 + i\epsilon\right]} u(q_1).$$

Once again let's introduce Feynman parameters:

$$\frac{1}{ABC} = 2\int_0^1 dx\,dy\,dz\,\delta(x+y+z-1)\frac{1}{[Ax+By+Cz]^3},$$

where $A \equiv k^2 - m_f^2 + i\epsilon$, $B \equiv (p+k)^2 - m_f^2 + i\epsilon$, $C \equiv (q_1-k)^2 - m_s^2 + i\epsilon$. One recovers the same A, B, C as in the previous loop calculation if $f \to s$ or vice versa. Therefore, the computation of the denominator is very similar, giving $(k^2 - \Delta + i\epsilon)^3$ after shifting $k^\mu \to k^\mu - yp^\mu + zq_1^\mu$, with Δ now defined as: $\boxed{\Delta \equiv -xyp^2 - z(1-z)m_\mu^2 + (1-z)m_f^2 + zm_s^2}$. The calculation of the numerator is a bit different though (once again the expressions below are implied to be sandwiched between the two external spinors).

The numerator:

$$\begin{aligned} N^\mu &= (1-\gamma_5)(\not{k}+\not{p}+m_f)\gamma^\mu(\not{k}+m_f)(1+\gamma_5) \\ &= (1-\gamma_5)\left(\not{k}\gamma^\mu\not{k} + \not{p}\gamma^\mu\not{k} + m_f\gamma^\mu\not{k} + m_f\not{k}\gamma^\mu + m_f\not{p}\gamma^\mu + m_f^2\gamma^\mu\right)(1+\gamma_5) \\ &= (\not{k}\gamma^\mu\not{k} + \not{p}\gamma^\mu\not{k})(1+\gamma_5)^2 + (m_f\gamma^\mu\not{k} + m_f\not{k}\gamma^\mu + m_f\not{p}\gamma^\mu)(1-\gamma_5)(1+\gamma_5) \\ &\quad + m_f^2\gamma^\mu(1+\gamma_5)^2 \\ &= 2(\not{k}\gamma^\mu\not{k} + \not{p}\gamma^\mu\not{k})(1+\gamma_5) + 2m_f^2\gamma^\mu(1+\gamma_5). \end{aligned}$$

Shifting $k^\mu \to k^\mu - yp^\mu + zq_1^\mu$ and neglecting the γ^μ term as we only need $\sigma^{\mu\nu}$ part of N^μ gives:

$$\begin{aligned} N^\mu &= 2\left\{(\not{k} - y\not{p} + z\not{q}_1)\gamma^\mu(\not{k} - y\not{p} + z\not{q}_1) + \not{p}\gamma^\mu(\not{k} - y\not{p} + z\not{q}_1)\right\}(1+\gamma_5) \\ &= 2\Big\{\not{k}\gamma^\mu\not{k} - y\,\cancelto{o}{\not{p}\gamma^\mu\not{k}} + z\,\cancelto{o}{\not{q}_1\gamma^\mu\not{k}} - y\,\cancelto{o}{\not{k}\gamma^\mu\not{p}} + y^2\not{p}\gamma^\mu\not{p} - yz\not{q}_1\gamma^\mu\not{p} + z\,\cancelto{o}{\not{k}\gamma^\mu\not{q}_1} \\ &\quad - yz\not{p}\gamma^\mu\not{q}_1 + z^2\not{q}_1\gamma^\mu\not{q}_1 + \cancelto{o}{\not{p}\gamma^\mu\not{k}} - y\not{p}\gamma^\mu\not{p} + z\not{p}\gamma^\mu\not{q}_1\Big\}(1+\gamma_5) \\ &= 2\left\{(y^2-y)\not{p}\gamma^\mu\not{p} - yz\not{q}_1\gamma^\mu\not{p} - yz\not{p}\gamma^\mu\not{q}_1 + z^2\not{q}_1\gamma^\mu\not{q}_1 + z\not{p}\gamma^\mu\not{q}_1\right\}(1+\gamma_5) \end{aligned}$$

$$
\begin{aligned}
&= 2\left\{(y^2-y)\not{q}_2\gamma^\mu\not{q}_2-(y^2-y)\not{q}_1\gamma^\mu\not{q}_2-(y^2-y)\not{q}_2\gamma^\mu\not{q}_1+(y^2-y)\not{q}_1\gamma^\mu\not{q}_1\right.\\
&\quad -yz\,\not{q}_1\gamma^\mu\not{q}_2+yz\,\not{q}_1\gamma^\mu\not{q}_1-yz\,\not{q}_2\gamma^\mu\not{q}_1+yz\,\not{q}_1\gamma^\mu\not{q}_1+z^2\,\not{q}_1\gamma^\mu\not{q}_1\\
&\quad \left.+z\,\not{q}_2\gamma^\mu\not{q}_1-z\not{q}_1\gamma^\mu\not{q}_1\right\}(1+\gamma_5)\\
&= 2\left\{(y^2-y)\not{q}_2\gamma^\mu\not{q}_2-\left(y^2-y+yz\right)\not{q}_1\gamma^\mu\not{q}_2+\left(z-yz-y^2+y\right)\not{q}_2\gamma^\mu\not{q}_1\right.\\
&\quad \left.+\left(2yz+z^2-z+y^2-y\right)\not{q}_1\gamma^\mu\not{q}_1\right\}(1+\gamma_5)\\
&= 2\left\{(y^2-y)m_\mu\gamma^\mu\not{q}_2-\left(y^2-y+yz\right)\not{q}_1\gamma^\mu\not{q}_2+\left(z-yz-y^2+y\right)m_\mu\gamma^\mu\not{q}_1\right.\\
&\quad \left.+\left(2yz+z^2-z+y^2-y\right)\not{q}_1\gamma^\mu\not{q}_1\right\}(1+\gamma_5)\,.
\end{aligned}
$$

$$
\begin{aligned}
\overline{u(q_2)}\gamma^\mu\not{q}_2\,(1+\gamma_5)\,u(q_1) &= \overline{u(q_2)}\,q_{2_\nu}\gamma^\mu\gamma^\nu\,(1+\gamma_5)\,u(q_1)\\
&= \overline{u(q_2)}\,2g^{\mu\nu}q_{2_\nu}\,(1+\gamma_5)\,u(q_1)-\overline{u(q_2)}\,q_{2_\nu}\gamma^\nu\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
&= \overline{u(q_2)}\,2q_2^\mu\,(1+\gamma_5)\,u(q_1)-\overline{u(q_2)}\,\not{q}_2\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
&= \overline{u(q_2)}\,\left\{2q_2^\mu\,(1+\gamma_5)-m_\mu\gamma^\mu\,(1+\gamma_5)\right\}\,u(q_1).
\end{aligned}
$$

$$
\begin{aligned}
\overline{u(q_2)}\,\not{q}_1\gamma^\mu\not{q}_2\,(1+\gamma_5)\,u(q_1) =&\ \overline{u(q_2)}\,q_{2_\nu}\not{q}_1\gamma^\mu\gamma^\nu\,(1+\gamma_5)\,u(q_1)\\
=&\ \overline{u(q_2)}\,2g^{\mu\nu}q_{2_\nu}\not{q}_1\,(1+\gamma_5)\,u(q_1)-\overline{u(q_2)}\,\not{q}_1\not{q}_2\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
=&\ \overline{u(q_2)}\,2m_\mu q_2^\mu\,(1-\gamma_5)\,u(q_1)-\overline{u(q_2)}\,2g^{\alpha\beta}q_{1_\alpha}q_{2_\beta}\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
&+\ \overline{u(q_2)}\,q_{1_\alpha}q_{2_\beta}\gamma^\beta\gamma^\alpha\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
=&\ \overline{u(q_2)}\,2m_\mu q_2^\mu\,(1-\gamma_5)\,u(q_1)-\overline{u(q_2)}\,2(q_1q_2)\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
&+\ \overline{u(q_2)}\,\not{q}_2\not{q}_1\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
=&\ \overline{u(q_2)}\,2m_\mu q_2^\mu\,(1-\gamma_5)\,u(q_1)-\overline{u(q_2)}\,2(q_1q_2)\gamma^\mu\,(1+\gamma_5)\,u(q_1)\\
&+\overline{u(q_2)}\,m_\mu 2g^{\mu\nu}q_{1_\nu}\,(1+\gamma_5)\,u(q_1)-\overline{u(q_2)}\,m_\mu q_{1_\nu}\gamma^\mu\gamma^\nu\,(1+\gamma_5)\,u(q_1)\\
=&\ \overline{u(q_2)}\,\left\{2m_\mu q_2^\mu\,(1-\gamma_5)-2(q_1q_2)\gamma^\mu\,(1+\gamma_5)\right.\\
&\left.+2m_\mu q_1^\mu\,(1+\gamma_5)-m_\mu^2\gamma^\mu\,(1-\gamma_5)\right\}\,u(q_1).
\end{aligned}
$$

$$
\overline{u(q_2)}\,\gamma^\mu\not{q}_1\,(1+\gamma_5)\,u(q_1) = \overline{u(q_2)}\,m_\mu\gamma^\mu\,(1-\gamma_5)\,u(q_1).
$$

$$
\begin{aligned}
\overline{u(q_2)}\,\not{q}_1\gamma^\mu\not{q}_1\,(1+\gamma_5)\,u(q_1) &= \overline{u(q_2)}\,m_\mu\,\not{q}_1\gamma^\mu\,(1-\gamma_5)\,u(q_1)\\
&= \overline{u(q_2)}\,2m_\mu\,g^{\mu\nu}q_{1_\nu}\,(1-\gamma_5)\,u(q_1)-\overline{u(q_2)}\,m_\mu\gamma^\mu\not{q}_1\,(1-\gamma_5)\,u(q_1)\\
&= \overline{u(q_2)}\,\left\{2m_\mu\,q_1^\mu\,(1-\gamma_5)-m_\mu^2\gamma^\mu\,(1+\gamma_5)\right\}\,u(q_1).
\end{aligned}
$$

Plugging the above results back in N^μ yields:

$$
N^\mu = 2\left\{(y^2-y)2m_\mu\,q_2^\mu\,(1+\gamma_5)-\left(y^2-y+yz\right)2m_\mu\,q_2^\mu\,(1-\gamma_5)\right.
$$

$$
\begin{aligned}
&- \left(y^2 - y + yz\right) 2m_\mu \, q_1^\mu \,(1+\gamma_5) + \left(2yz + z^2 - z + y^2 - y\right) 2m_\mu \, q_1^\mu \,(1-\gamma_5)\big\} \\
&+ (\gamma^\mu \, \text{terms})\,.
\end{aligned}
$$

Some factors can be rewritten in the following way:

- $z^2 - z + 2yz = z(z-1) + 2yz = -z(x+y) + 2yz = -xz - yz + 2yz = yz - xz = \cancelto{0}{(y-x)z}$.
- $y^2 - y = y(y-1) = -y(x+z) = -xy - yz$.

$$
\begin{aligned}
\Longrightarrow \quad N^\mu = & \, 2\left\{(y^2-y)2m_\mu \, q_2^\mu \,(2\gamma_5) - (yz)2m_\mu \, q_2^\mu \,(1-\gamma_5)\right. \\
& \left. + \left(y^2 - y\right) 2m_\mu \, q_1^\mu \,(-2\gamma_5) - (yz)2m_\mu \, q_1^\mu \,(1+\gamma_5)\right\} \\
= & \, 8(y^2-y)m_\mu \, p^\mu \gamma_5 + 4yz \, m_\mu \, p^\mu \gamma_5 - \boxed{4yz \, m_\mu \, (q_2^\mu + q_1^\mu)}\,.
\end{aligned}
$$

The last term is exactly what we need; It gives the $\sigma^{\mu\nu}$ term after applying the Gordon identity:

$$
\overline{u(q_2)} \,\{-4yz \, m_\mu \,(q_2^\mu + q_1^\mu)\} \, u(q_1) = \overline{u(q_2)} \,\left\{-8yz \, m_\mu^2 \gamma^\mu + 4yz \, m_\mu i \sigma^{\mu\nu} p_\nu\right\} u(q_1).
$$

p^μ terms in N^μ vanish due to the Ward identity. Finally the contribution to $F(0)$ is:

$$
\begin{aligned}
F_2(0) = & \frac{2m_\mu}{e}\left(-Q_f e \frac{|\lambda|^2}{4}\right) 2 \int_0^1 dx \, dy \, dz \, \delta(x+y+z-1) \int\limits_{-\infty}^{\infty} \frac{d^4k}{(2\pi)^4} \frac{4yz \, m_\mu i}{(k^2 - \Delta + i\epsilon)^3} \\
= & \left(-Q_f |\lambda|^2\right) 4m_\mu^2 \int_0^1 dx \, dy \, dz \, \delta(x+y+z-1) \, yzi \left[\frac{-i}{32\pi^2\Delta}\right] \\
= & \left(-Q_f |\lambda|^2\right) \frac{m_\mu^2}{8\pi^2} \int_0^1 dz \, \frac{z}{\Delta} \int_0^{1-z} dy \, y \\
= & \left(-Q_f |\lambda|^2\right) \frac{m_\mu^2}{8\pi^2} \int_0^1 dz \, \frac{z(1-z)^2}{2\Delta}.
\end{aligned}
$$

Change of variables: $z = 1 - x \implies x = 1 - z$ & $dz = -dx$. In terms of x $F_2(0)$ becomes:

$$
F_2(0) = -Q_f |\lambda|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^2 - x^3}{\Delta}\,, \tag{6.138}
$$

where $\Delta = m_\mu^2 x^2 + (m_f^2 - m_\mu^2)x + m_s^2(1-x)$.

With the loop calculations performed and the results (6.137) and (6.138) obtained, I will now calculate $\not{R}_p$ contributions to a_μ from the diagrams in Fig. 16. Using these equations and neglecting muon and lepton masses we get:

$$
a_\mu^{(a)} = |\lambda_{32k}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^2 - x^3}{m_\mu^2 x^2 + (m_{\ell_k}^2 - m_\mu^2)x + m_{\tilde{\nu}_\tau}^2 (1-x)}
$$

$$\approx |\lambda_{32k}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^2(1-x)}{m_{\widetilde{\nu}_\tau}(1-x)} = \frac{|\lambda_{32k}|^2}{48\pi^2} \frac{m_\mu^2}{m_{\widetilde{\nu}_\tau}^2}, \tag{6.139}$$

$$a_\mu^{(b)} = |\lambda_{i23}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^3 - x^2}{m_\mu^2 x^2 + (m_{\widetilde{\tau}_R}^2 - m_\mu^2)x + m_{\nu_i}^2(1-x)}$$

$$\approx |\lambda_{i23}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^3 - x^2}{m_{\widetilde{\tau}_R}^2 x} = -\frac{|\lambda_{i23}|^2}{96\pi^2} \frac{m_\mu^2}{m_{\widetilde{\tau}_R}^2}, \tag{6.140}$$

$$a_\mu^{(c)} = |\lambda_{3j2}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^2 - x^3}{m_\mu^2 x^2 + (m_{\ell_j}^2 - m_\mu^2)x + m_{\widetilde{\nu}_\tau}^2(1-x)}$$

$$\approx \frac{|\lambda_{3j2}|^2}{48\pi^2} \frac{m_\mu^2}{m_{\widetilde{\nu}_\tau}^2}, \tag{6.141}$$

$$a_\mu^{(d)} = |\lambda_{i32}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^3 - x^2}{m_\mu^2 x^2 + (m_{\widetilde{\tau}_L}^2 - m_\mu^2)x + m_{\nu_i}^2(1-x)}$$

$$\approx -\frac{|\lambda_{i32}|^2}{96\pi^2} \frac{m_\mu^2}{m_{\widetilde{\tau}_L}^2}. \tag{6.142}$$

Altogether the λ contributions to the muon anomalous magnetic moment a_μ^λ [16] is

$$a_\mu^\lambda = \frac{m_\mu^2}{96\pi^2} \left(|\lambda_{32ki}|^2 \frac{2}{m_{\widetilde{\nu}_\tau}^2} - \frac{|\lambda_{i23}|^2}{m_{\widetilde{\tau}_R}^2} + |\lambda_{3i2}|^2 \left[\frac{2}{m_{\widetilde{\nu}_\tau}^2} - \frac{1}{m_{\widetilde{\tau}_L}^2} \right] \right). \tag{6.143}$$

Similar calculations for the λ' contribution can also be performed. In particular, the evaluation of $a_\mu^{(e)}$ and $a_\mu^{(h)}$ is straightforward:

$$a_\mu^{(e)} = 3\left(\frac{1}{3}\right) |\lambda'_{23k}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^2 - x^3}{m_\mu^2 x^2 + (m_{d_k}^2 - m_\mu^2)x + m_{\widetilde{t}_L}^2(1-x)}$$

$$\approx \frac{|\lambda'_{23k}|^2}{48\pi^2} \frac{m_\mu^2}{m_{\widetilde{t}_L}^2}, \tag{6.144}$$

$$a_\mu^{(h)} = 3\left(\frac{2}{3}\right) |\lambda'_{23k}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^3 - x^2}{m_\mu^2 x^2 + (m_{\widetilde{t}_L}^2 - m_\mu^2)x + m_{d_k}^2(1-x)}$$

$$\approx -\frac{|\lambda'_{23k}|^2}{48\pi^2} \frac{m_\mu^2}{m_{\widetilde{t}_L}^2}. \tag{6.145}$$

Thus $a_\mu^{(e)}$ and $a_\mu^{(h)}$ cancel each other. As for the two remaining diagrams, I will still neglect m_μ, but leave m_{u_j} untouched, since for $j = 3$ u is a top quark t and its mass can be comparable to the sparticle mass involved.

$$a_\mu^{(f)} = 3\left(\frac{1}{3}\right) |\lambda'_{2j3}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \, \frac{x^3 - x^2}{m_\mu^2 x^2 + (m_{\widetilde{b}_R}^2 - m_\mu^2)x + m_{u_j}^2(1-x)}$$

$$\approx |\lambda'_{2j3}|^2 \frac{m_\mu^2}{16\pi^2} \int_0^1 dx \frac{x^3 - x^2}{m_{\widetilde{b}_R}^2 x + m_{u_j}^2 - m_{u_j}^2 x}.$$

$$\int_0^1 dx \frac{x^3 - x^2}{(m^2_{\tilde{b}_R} - m^2_{u_j})x + m^2_{u_j}} = \int_0^1 dx \frac{x^3 - x^2}{ax + b}, \text{ where } a \equiv m^2_{\tilde{b}_R} - m^2_{u_j} \text{ and } b \equiv m^2_{u_j}.$$

$$\begin{aligned}
\int_0^1 dx \frac{x^3 - x^2}{ax + b} &= \frac{a(-a^2 + 3ab + 6b^2) + 6b^2(a+b)\ln\left(\frac{b}{a+b}\right)}{6a^4} \\
&= -\frac{1}{6a}\left[1 - \frac{3b}{a} - \frac{6b^2}{a^2} - \frac{6b^2(a+b)\ln\left(\frac{b}{a+b}\right)}{a^3}\right] \\
&= -\frac{1}{6a}\left[1 + \frac{6b}{a}\left\{-\frac{1}{2} - \frac{b}{a} - \frac{b(a+b)\ln\left(\frac{b}{a+b}\right)}{a^2}\right\}\right] \\
&= -\frac{1}{6a}\left[1 + \frac{6m^2_{u_j}}{m^2_{\tilde{b}_R}\left(1 - \frac{m^2_{u_j}}{m^2_{\tilde{b}_R}}\right)}\left\{-\frac{1}{2} - \frac{m^2_{u_j}}{m^2_{\tilde{b}_R}\left(1 - \frac{m^2_{u_j}}{m^2_{\tilde{b}_R}}\right)} - \frac{m^2_{u_j} m^2_{\tilde{b}_R} \ln\left(\frac{m^2_{u_j}}{m^2_{\tilde{b}_R}}\right)}{m^4_{\tilde{b}_R}\left(1 - \frac{m^2_{u_j}}{m^2_{\tilde{b}_R}}\right)^2}\right\}\right] \\
&= -\frac{1}{6a}\left[1 + \frac{6y}{1-y}\left\{-\frac{1}{2} - \frac{y}{1-y} - \frac{y\ln y}{(1-y)^2}\right\}\right] = -\frac{1}{6a}\left[1 + C_1(y)\right],
\end{aligned}$$

where

$$C_1(y) \equiv \frac{6y}{1-y}\left\{\frac{1}{2} - \frac{1}{1-y} - \frac{y\ln y}{(1-y)^2}\right\}, \qquad (6.146)$$

as defined in Ref. [16] and $y \equiv \frac{m^2_{u_j}}{m^2_{\tilde{b}_R}}$. Using the above result, it follows that

$$a_\mu^{(f)} = -\frac{|\lambda'_{2j3}|^2}{96\pi^2}\frac{m^2_\mu}{m^2_{\tilde{b}_R} - m^2_{u_j}}(1 + C_1(y)). \qquad (6.147)$$

$$\begin{aligned}
a_\mu^{(g)} &= 3\left(\frac{2}{3}\right)|\lambda'_{2j3}|^2 \frac{m^2_\mu}{16\pi^2}\int_0^1 dx \, \frac{x^2 - x^3}{m^2_\mu x^2 + (m^2_{u_j} - m^2_\mu)x + m^2_{\tilde{b}_R}(1-x)} \\
&\approx |\lambda'_{2j3}|^2 \frac{m^2_\mu}{8\pi^2}\int_0^1 dx \frac{x^3 - x^2}{-m^2_{u_j}x + m^2_{\tilde{b}_R}x - m^2_{\tilde{b}_R}}.
\end{aligned}$$

$$\int_0^1 dx \frac{x^3 - x^2}{(m^2_{\tilde{b}_R} - m^2_{u_j})x - m^2_{\tilde{b}_R}} = \int_0^1 dx \frac{x^3 - x^2}{ax - b}, \text{ where } a \equiv m^2_{\tilde{b}_R} - m^2_{u_j} \text{ and } m^2_{\tilde{b}_R}.$$

$$\int_0^1 dx \frac{x^3 - x^2}{ax - b} = \frac{-a(a^2 + 3ab - 6b^2) + 6b^2(-a+b)\ln\left(1 - \frac{a}{b}\right)}{6a^4}$$

By simplifying the above expression and defining $y \equiv \frac{m_{u_j}^2}{m_{\tilde{b}_R}^2}$ as in the $a_\mu^{(f)}$ case the following result is obtained:

$$\int_0^1 dx \frac{x^3 - x^2}{ax - b} = \frac{1}{3a}\left[1 + \frac{3y}{1-y}\left\{\frac{1}{2} + \frac{1}{1-y} + \frac{\ln y}{(1-y)^2}\right\}\right] = \frac{1}{3a}\left[1 + C_2(y)\right],$$

where [16]

$$C_2(y) \equiv \frac{3y}{1-y}\left\{\frac{1}{2} + \frac{1}{1-y} + \frac{\ln y}{(1-y)^2}\right\}, \tag{6.148}$$

Finally $a_\mu^{(g)}$ is

$$a_\mu^{(g)} = \frac{|\lambda'_{2j3}|^2}{24\pi^2}\frac{m_\mu^2}{m_{\tilde{b}_R}^2 - m_{u_j}^2}(1 + C_2(y)). \tag{6.149}$$

$C_1(y)$ and $C_2(y)$ are only relevant when $j = 3$. For the first two generations they can be neglected as $y \ll 1$ in those cases. Combining (6.147) and (6.149) then yields the following total contribution to a_μ coming from the λ' couplings [16]:

$$\begin{aligned} a_\mu^{\lambda'} =& \frac{m_\mu^2}{32\pi^2}\left\{\frac{1}{m_{\tilde{b}_R}^2 - m_t^2}|\lambda'_{233}|^2\left(1 + \frac{2y_t}{1-y_t}\left[\frac{1}{2} + \frac{3}{1-y_t} + \frac{2+y_t}{(1-y_t)^2}\ln y_t\right]\right)\right\} \\ &+ \frac{m_\mu^2}{32\pi^2}\left\{\frac{1}{m_{\tilde{b}_R}^2}(|\lambda'_{213}|^2 + |\lambda'_{223}|^2)\right\}. \end{aligned} \tag{6.150}$$

Note that $a_\mu^{\lambda'}$ is positive definite. The total contribution $\Delta a_\mu^{\not{R}_p}$ [16] in the ESUSY with R_p violation is then

$$\Delta a_\mu^{\not{R}_p} = a_\mu^\lambda + a_\mu^{\lambda'}. \tag{6.151}$$

The theoretical and experimental values of a_μ are [6]

$$\begin{aligned} a_\mu^{SM} &= (1165918.36 \pm 0.44) \times 10^{-9}, \\ a_\mu^{exp} &= (1165920.91 \pm 0.63) \times 10^{-9}. \end{aligned}$$

Using the above numbers and setting, for simplicity, $m_{\tilde{\nu}_\tau}^2 = m_{\tilde{\tau}_L}^2 = m_{\tilde{\tau}_R}^2 = m_{\tilde{b}_R}^2 \equiv \widetilde{m}$ [16], I get the following lower bound on the couplings λ, λ' at 1σ (2σ) level:

$$|\lambda| \ (\text{or}|\lambda'|) > 0.72 \ (0.54) \times \left(\frac{\widetilde{m}}{100\,\text{GeV}}\right). \tag{6.152}$$

This means that we can neglect contributions coming from the λ' couplings since there are unavoidable single bounds from other processes (see Ref. [16] or Ref. [3] and also the bounds coming from $R_{\tau\pi}$ and D meson decays in previous sections). With all the masses set to $\widetilde{m}$ a_μ^λ then becomes [16]:

$$a_\mu^\lambda \approx \frac{m_\mu^2}{96\pi^2\,\widetilde{m}^2}\left(2|\lambda_{321}|^2 + 3|\lambda_{322}|^2 + |\lambda_{323}|^2 + |\lambda_{312}|^2 - |\lambda_{123}|^2\right). \tag{6.153}$$

First of all we can neglect λ_{123} contribution since there are stringent bounds on it from the CC universality. Even if it were not the case, the λ_{123} contribution is negative, so no stringent bound could be obtained from equation (6.153). The same applies to λ_{312} and λ_{321} couplings since they are constrained by asymmetries in e^+e^- collisions [16]. The remaining two couplings are the least constrained of all the coupling constants involved, especially the coupling constant λ_{322} (see the bounds obtained in previous sections), which in the ESUSY can satisfy eq. (6.152) due to the high mass of smuons. Following the Ref. [16], we can treat this coupling as the dominant one. $\Delta a_\mu^{\not{R}_p}$ then becomes:

$$\Delta a_\mu^{\not{R}_p} \approx 3.48 \times 10^{-9} \left(\frac{100\,\text{GeV}}{\widetilde{m}}\right)^2 |\lambda_{322}|^2. \tag{6.154}$$

From the above equation I get the following 1σ level bound:

$$|\lambda_{322}| = 0.86 \begin{array}{c} +0.12 \\ -0.14 \end{array} \left(\frac{\widetilde{m}}{100\,\text{GeV}}\right). \qquad 1\sigma \tag{6.155}$$

So the λ_{322} coupling can in principle account for the extra contribution to the anomalous magnetic dipole moment of muon.

7 Conclusion

To conclude I summarize my results in Table 4. In the second column of Table 4 various observables are listed. Following the notation of Ref. [3], V_{ud} stands for the CKM martix unitarity, while observables R denote various ratios of branching fractions in the CC sector (see section 6 for the exact definitions). A_{FB} and $Q_W(CS)$ are used for forward-backward asymmetry in e^+e^- collisions and atomic parity violation in Cs atoms. Finally $\nu_\mu e$ and $\nu_\mu q$ denote muon-neutrino - electron elastic scattering and muon-neutrino - nucleon deep inelastic scattering respectively. The third column lists my bounds obtained in section 6, while the last two columns contain previous bounds from Ref.s [3] (2004 review of R-parity violating SUSY by Barbier et al.) and [7] (2009 paper by Y. Kao and T. Takeuchi, where some of the bounds got updated). All of these bounds are 2σ level bounds. A handy notation [3] like $0.02\widetilde{\ell}_{Rk}$ used in Table 4 represents in this case a concise version of a numerical relationship $\lambda_{ijk} < 0.02 \left(\frac{m_{\widetilde{\ell}_{Rk}}}{100\text{GeV}}\right)$.

As can be observed in Table 4 the CC sector gives the most stringent bounds and has experienced a steady improvement; almost all the bounds from the CC observables have gradually improved throughout the last 20-30 years. In most cases I got stronger bounds by at least about 20%. The bounds from other observables fluctuate, but most of them seem to stay in roughly 25%-50% range. Hopefully with the reduction of experimental uncertainties, these bounds will also become more stringent.

Coupling	Observable	My 2σ bounds	Previous 2σ bounds [3]	Prev. 2σ bounds [7]
λ_{12k}	V_{ud}	$0.02\,\widetilde{\ell}_{Rk}$	$0.05\,\widetilde{\ell}_{Rk}$	$0.03\,\widetilde{\ell}_{Rk}$
	$R_{\tau\mu}$	$0.01\,\widetilde{\ell}_{Rk}$	$0.07\,\widetilde{\ell}_{Rk}$	$0.05\,\widetilde{\ell}_{Rk}$
	$Q_W(Cs)$	$0.10\,\widetilde{\ell}_{Rk}$	$0.11\,\widetilde{\ell}_{Rk}$	$0.08\,\widetilde{\ell}_{Rk}$
	$\nu_\mu e$	$0.14\,\widetilde{\ell}_{Rk}$	$0.14\,\widetilde{\ell}_{Rk}$	
	$\nu_\mu e$	$0.13\,\widetilde{\ell}_{Lk=1}$	$0.13\,\widetilde{\ell}_{Lk=1}$	
	$\nu_\mu q$	$0.25\,\widetilde{\ell}_{Rk}$	$0.13\,\widetilde{\ell}_{Rk}$	
	$A_{FB}\,[k\,=\,1,2,3]$	$[0.36,\ 0.19,\ 0.13]\widetilde{\nu}$	$[0.37,\ 0.25,\ 0.11]\widetilde{\nu}$	
λ_{13k}	R_τ	$0.03\,\widetilde{\ell}_{Rk}$	$0.07\,\widetilde{\ell}_{Rk}$	$0.05\,\widetilde{\ell}_{Rk}$
	$A_{FB}\,[k\,=\,1,2,3]$	$[0.36,\ 0.19,\ 0.13]\widetilde{\nu}$	$[0.37,\ 0.25,\ 0.11]\widetilde{\nu}$	
λ_{23k}	R_τ	$0.06\,\widetilde{\ell}_{Rk}$	$0.07\,\widetilde{\ell}_{Rk}$	$0.05\,\widetilde{\ell}_{Rk}$
	$R_{\tau\mu}$	$0.05\,\widetilde{\ell}_{Rk}$	$0.07\,\widetilde{\ell}_{Rk}$	$0.06\,\widetilde{\ell}_{Rk}$
	$\nu_\mu e,\ k=1$	$0.12\,\widetilde{\tau}_L$	$0.11\,\widetilde{\tau}_L$	
λ'_{11k}	V_{ud}	$0.01\,\widetilde{d}_{Rk}$	$0.02\,\widetilde{d}_{Rk}$	$0.03\,\widetilde{d}_{Rk}$
	R_π	$0.02\,\widetilde{d}_{Rk}$	$0.03\,\widetilde{d}_{Rk}$	$0.03\,\widetilde{d}_{Rk}$
	$Q_W(Cs)$	$0.05\,\widetilde{d}_{Rk}$	$0.05\,\widetilde{d}_{Rk}$	$0.04\,\widetilde{d}_{Rk}$
	$A_{FB}\,[k\,=\,2,3]$	$[0.30,\ = 0.10\ {}^{+0.04}_{-0.08}\ ]\widetilde{u}_L$	$[0.28,\ 0.18]\widetilde{u}_L$	
λ'_{12k}	R_{D^0}	$0.11\,\widetilde{d}_{Rk}$	$0.27\,\widetilde{d}_{Rk}$	$0.2\,\widetilde{d}_{Rk}$
	R_{D^+}	$0.08\,\widetilde{d}_{Rk}$	$0.44\,\widetilde{d}_{Rk}$	$0.2\,\widetilde{d}_{Rk}$
	$R^*_{D^+}$	$0.11\,\widetilde{d}_{Rk}$	$0.23\,\widetilde{d}_{Rk}$	$0.2\,\widetilde{d}_{Rk}$
	A_{FB}	$0.21\,\widetilde{d}_{Rk}$	$0.21\,\widetilde{d}_{Rk}$	
	$A_{FB}\,[k\,=\,2,3]$	$[0.30,\ = 0.10\ {}^{+0.04}_{-0.08}\ ]\widetilde{c}_L$	$[0.28,\ 0.18]\widetilde{c}_L$	
λ'_{21k}	R_π	$0.06\,\widetilde{d}_{Rk}$	$0.06\,\widetilde{d}_{Rk}$	$0.06\,\widetilde{d}_{Rk}$
	$R_{\tau\pi}$	$0.07\,\widetilde{d}_{Rk}$	$0.08\,\widetilde{d}_{Rk}$	$0.07\,\widetilde{d}_{Rk}$
	$\nu_\mu q$	$0.25\,\widetilde{d}_{Rk}$	$0.15\,\widetilde{d}_{Rk}$	
λ'_{22k}	R_{D^0}	$0.08\,\widetilde{d}_{Rk}$	$0.21\,\widetilde{d}_{Rk}$	$0.1\,\widetilde{d}_{Rk}$
	R_{D^+}	$0.16\,\widetilde{d}_{Rk}$	$0.61\,\widetilde{d}_{Rk}$	$0.4\,\widetilde{d}_{Rk}$
	$R^*_{D^+}$	$0.18\,\widetilde{d}_{Rk}$	$0.38\,\widetilde{d}_{Rk}$	$0.3\,\widetilde{d}_{Rk}$
	$R_{D_s}(\tau\mu)$	$0.18\,\widetilde{d}_{Rk}$	$0.65\,\widetilde{d}_{Rk}$	$0.2\,\widetilde{d}_{Rk}$
λ'_{31k}	$R_{\tau\pi}$	$0.04\,\widetilde{d}_{Rk}$	$0.12\,\widetilde{d}_{Rk}$	$0.06\,\widetilde{d}_{Rk}$
λ'_{32k}	$R_{D_s}(\tau\mu)$	$0.21\,\widetilde{d}_{Rk}$	$0.52\,\widetilde{d}_{Rk}$	$0.3\,\widetilde{d}_{Rk}$
λ'_{1j1}	$Q_W(Cs)$	$0.03\,\widetilde{u}_{Lj}$	$0.03\,\widetilde{u}_{Lj}$	$0.03\,\widetilde{u}_{Lj}$
λ'_{2j1}	$\nu_\mu q$	$0.20\,\widetilde{d}_{Lj}$	$0.18\,\widetilde{d}_{Lj}$	

Table 4: My single-coupling bounds on various $\not{R}_p$ trilinear couplings compared to previous bounds of Ref.s [3] and [7].

References

[1] H.K. Dreiner, H.E. Haber, and S.P. Martin, "Two-component spinor techniques and Feynman rules for quantum field theory and supersymmetry", Phys. Rept. 494, 1 (2010) [hepph/ 0812.1594].

[2] Stephen P. Martin, *A Supersymmetry Primer*, arXiv:hep-ph/9709356v7.

[3] R. Barbier et al., Phys. Rept. 420, 1 (2005) [arXiv:hep-ph/0406039].

[4] V. Barger, G.F. Giudice, and T. Han, Phys. Rev. D40 (1989) 2987.

[5] Matthew D. Schwartz, "Quantum Field Theory and the Standard Model", Cambridge University Press, 2014.

[6] M. Tanabashi *et al.* [Particle Data Group], Phys. Rev. D **98**, 030001 (2018) and 2019 update.

[7] Y. Kao and T. Takeuchi, arXiv:0910.4980v2 [hep-ph].

[8] Halzen, F., and Martin, A. D. 1984. *Quarks and Leptons: An Introductory Course in Modern Particle Physics*. New York: John Wiley & Sons.

[9] R. Decker and M. Finkemeier, Nucl. Phys. B 438, 17 (1995) [hep-ph/9403385].

[10] J.H. Kuhn, E.Mirkes, and M. Finkemeier, arXiv:hep-ph/9511268v1.

[11] Marina Artuso, Brian Meadows, Alexey Petrov, Charm Meson Decays, Ann. Rev. Nucl. Part. Sci.58:249-291, 2008 (arXiv:0802.2934v1).

[12] Adeva B, Anderhub H, Ansari S, et al. Electroweak studies in e^+e^- collisions: $12 < sqrts < 46.78$ GeV. Phys Rev D Part Fields. 1988;38(9):2665-2678. doi:10.1103/physrevd.38.2665 .

[13] C.S. Wood et al., Science, 275, (1997) 1759.

[14] C. Wieman and A. Derevianko, [arXiv:1904.00281v1 [physics.atom-ph]].

[15] P.A. Zyla *et al.* [Particle Data Group], to be published in Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[16] J. E. Kim, B. Kyae and H. M. Lee, Phys. Lett. B520 (2001) 298.

[17] A. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B388 (1996) 588.