Introduction to physics of quarks with flavour and colour

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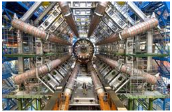
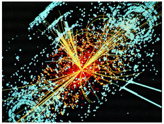
Today's landscape of particle physics

- proton-proton collisions at Large Hadron Collider

CERN (Switzerland, France),



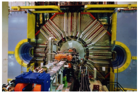
ATLAS detector



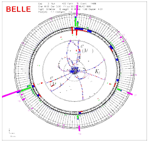
event image

- electron-positron collisions

KEK Laboratory (Japan)



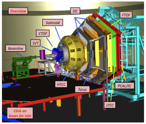
Belle detector



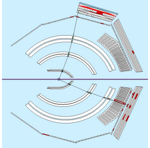
event reconstruction

- electron-nucleon collisions

Jefferson Laboratory (USA)



CLAS detector

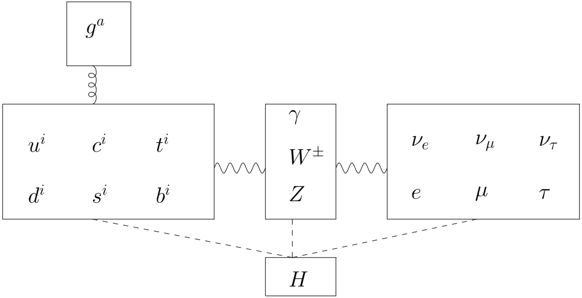


event reconstruction

- Results of experiments are translated in the language of theory
- Standard Model of elementary particles and their interactions

$i = 1, 2, 3,$

$a = 1, 2, \dots, 8$



- 3 families of 6 quarks and 6 leptons,
  each one with its flavour quantum number, quarks have colour charge
- flavourdynamics: theory of electroweak interactions,
  transmitted by $\gamma, W^\pm, Z, H$
- chromodynamics: theory of quark-gluon interactions,
  transmitted by gluons g^a

Plan of the lectures

- Lecture 1:

  Quarks and their quantum numbers,
  flavourdynamics

- Lecture 2:

  Calculating the colour-charge interaction
  in quantum chromodynamics

- Lecture 3:

  Determination of quark masses
  and mixing parameters

Quark flavours in the “ordinary “ matter

- proton and neutron are bound states of quarks
  ("strong" colour-charge interaction, see lecture 2):

  $|proton\rangle = |uud\rangle$, $\quad |neutron\rangle = |udd\rangle$

- quarks have spin 1/2,
  and electric charges $Q_u = +\frac{2}{3}e$, $Q_d = -\frac{1}{3}e$
  in the units of electron electric charge $Q_e = -e$

- u-quark and d-quark differ from each
  each quark type has its own flavour quantum number

- proton and neutron have different flavour content

The “ordinary” matter has three flavours

- protons & neutrons $\Rightarrow$ nuclei

  $|deuteron\rangle = |(uud)(udd)\rangle$ ("quark molecule")

- nuclei & electrons $\Rightarrow$ atoms $\Rightarrow$" molecules $\Rightarrow$"ordinary" matter (electromagnetic force, Coulomb potential)

- The electron (lepton) e has its own flavour quantum number

- “ordinary” (atomic/molecular) matter involves three flavours: two quark flavours and one lepton flavour:

  $\{\, u, d, e \,\}$

- experiment: strong interaction of the quarks within nucleon and the electromagnetic interactions obey conservation of each flavour

Flavour in quantum field theory

- quarks and electrons with spin 1/2 (fermions) are described by Dirac equation:

$$(i\partial_\mu \gamma^\mu - m)\psi(x) = 0$$

free fermion with mass m and
4-component wave function (bispinor)

$$\psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \\ \psi_3(x) \\ \psi_4(x) \end{pmatrix}$$

$x^\mu = (x_0, \vec{x})$ - 4-dim. coordinates, $\partial_\mu = \partial/\partial x_\mu$, γ_μ -Dirac matrices:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \ \vec{\gamma} = \{\gamma^1, \gamma^2, \gamma^3\} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

- in particle physics we use the system of units where the Plank constant $\hbar$ =1 and velocity of light $c = 1$, $x_0 = t$
- this equation describes both fermion and antifermion: antiquarks $\overline{u}$, $\overline{d}$ and positrons e^+ (antimatter)

Flavour in quantum field theory

- Dirac equation derived as "equation of motion" (Euler-Lagrange) for the fields $\psi(x), \overline{\psi}(x)$, from the principle of least action:

$$\mathcal{S} = \int d^4x \mathcal{L}(x)$$

- Lagrangian $\mathcal{L}(x)$ describing three-flavour matter (no interactions)

$$\begin{aligned} \mathcal{L}^{(u,d,e)}(x) &= \overline{\psi}_u(x)(i\partial_\mu\gamma^\mu - m_u)\psi_u(x) + \overline{\psi}_d(x)(i\partial_\mu\gamma^\mu - m_d)\psi_d(x) \\ &+ \overline{\psi}_e(x)(i\partial_\mu\gamma^\mu - m_e)\psi_e(x) \end{aligned}$$

$x_\mu = (x_0, \vec{x})$ - 4-dim. coordinate, $\partial_\mu = \partial/\partial x_\mu$, $\gamma_\mu = (\gamma_0, \vec{\gamma})$
$\overline{\psi} = \psi^+\gamma_0$ - the conjugated field

Symmetries of the Lagrangian

- space-time symmetry transformations
  (rotations, translations, Lorentz-transformations of x_μ)

  $\Rightarrow$ Lagrangian invariant
  $\Rightarrow$ conservation of angular momentum, spin, 4-momentum

- global gauge transformation, e.g., only for the u-quark field:

$$\begin{aligned}\psi_u(x) \to \psi'_u(x) &= \exp[-i\alpha_u]\,\psi_u(x)\,,\\ \overline{\psi}_u(x) \to \overline{\psi}'_u(x) &= \overline{\psi}_u(x)\exp[i\alpha_u]\,,\quad \alpha_u = const\end{aligned}$$

  the ψ_d and ψ_e fields do not transform, Lagrangian invariant

$$\mathcal{L}^{(u,d,e)}(x) \to \mathcal{L}^{(u,d,e)}(x)$$

- the set of these gauge transformations form a group $U(1)$
  product of elements, inverse element, unit element,...

Flavour quantum number

- bilinear combination of u quark fields: $j_\mu^{(u)} = \overline{\psi}_u \gamma_\mu \psi_u(x)$,

  global gauge invariance of $\mathcal{L}$ $\Rightarrow$ $\partial_\mu j_\mu^{(u)}(x) = 0$,

  u-flavour "charge" $\mathcal{U}(t) = \int d^3x\, j_0^{(u)}(t, \vec{x})$ conserved: $\frac{d}{dt}\mathcal{U}(t) = 0$

- resembles electrodynamics of electrons and positrons?:
  electromagnetic 4-dim. current density, conservation of electric charge

- u-flavour , d-flavour and e^--flavour are conserved quantum numbers, related to global gauge invariance

- flavours of antiquarks ($\overline{u}$ or $\overline{d}$) and antileptons (positron e^+-flavour) have opposite signs

- what happens when the interactions are "switched on" ?

Electromagnetic interaction of quarks and leptons

- 3-flavour matter (u, d, e^-) + photon (γ) = quantum electrodynamics,

$$\mathcal{L}_{QED}(x) = -\frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) + \sum_{f=u,d,e}\bar{\psi}_f(x)(iD^f_\mu\gamma^\mu - m_f)\psi_f(x)\,,$$

$$F_{\mu\nu} = \frac{\partial A_\nu(x)}{\partial x_\mu} - \frac{\partial A_\mu(x)}{\partial x_\nu}, \quad D^f_\mu = \partial_\mu - ieQ_f A_\mu(x)$$

- gauge transformation: the phase is an arbitrary function of x

$$\psi_f(x) \;\to\; \psi'_f(x) = \exp\left[-iQ_f\alpha(x)\right]\psi_f(x)\,,$$

$$\bar{\psi}_f(x) \;\to\; \bar{\psi}'_f(x) = \bar{\psi}_f(x)\exp\left[iQ_f\alpha(x)\right]\,, \quad f = u, d, e$$

$$A_\mu \;\to\; A'_\mu(x) = A_\mu(x) + \frac{1}{Q_f e}\partial_\mu\alpha(x)\,,$$

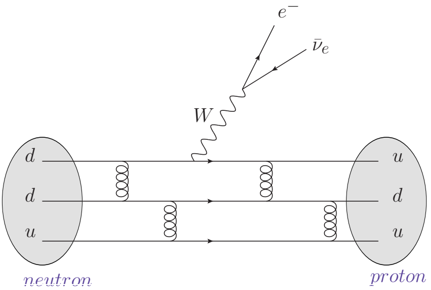
e-fundamental coupling, $Q_u = +2/3$, $Q_d = -1/3$, $Q_e = -1$

- electric charge conservation: $\alpha = const$,
  e.m. current $j^{(em)}_\mu = \sum_{f=u,d,e} Q_f\overline{\psi}_f\gamma_\mu\psi_f(x)$

- flavours are conserved in electromagnetic interactions

Flavour changing transitions

- neutron decays into proton in about 900 s, $n \to p\, e^- \bar{\nu}_e$, β-decay
- electron neutrino ν_e produced, own flavour quantum number



- at the quark level: $d \to u\, e^- \bar{\nu}_e$
- flavour changing transition transmitted by virtual W-boson

Flavour changing currents

- quark transition current: $j_\mu^{ud}(x) = V_{ud}\,\overline{u}(x)\gamma_\mu(1-\gamma_5)d(x)$
- V_{ud} a fundamental parameter of quark flavour physics
- lepton transition current: $j_\mu^{e\nu}(x) = \overline{e}(x)\gamma_\mu(1-\gamma_5)\nu_e(x)$
- $d \to ue\overline{\nu}_e$ a combination of two elementary interactions:

  $d \to uW$: $\mathcal{L}^{udW}(x) = g\, j_\mu^{ud} W^\mu$, g -universal coupling $\sim e$

  $W \to e\overline{\nu}_e$: $\mathcal{L}^{e\nu W}(x) = g\, j_\mu^{e\nu} W^\mu$
- new interaction terms $\mathcal{L}^{udW}$, $\mathcal{L}^{e\nu W}$, in the Lagrangian of Standard Model (in addition to $\mathcal{L}_{QED}$)

Weak interaciton

- the effective four-fermion interaction:

  $\int d^4x\, j^{e\nu}_\mu(x) W^\mu(x) W_\lambda(0) j^{ud,\lambda}(0) \simeq G_F j^{e\nu}_\mu(0) j^{ud,\mu}(0)$

  propagator of W-boson , $m_W \sim 80$ GeV » other energy scales,

  $\langle 0|W_\mu(x)W_\lambda(0)|0\rangle \sim \delta_{\mu\lambda}\delta^{(4)}(x)/m_W^2$, Fermi constant $g^2/m_W^2 = G_F$

- $n \to pe\bar{\nu}_l$ measured decay width ($\sim$ inverse lifetime):

$$\Gamma(n \to p\ell\nu_\ell) = |V_{ud}|^2 G_F^2 |\langle proton|j^{ud}_\mu|neutron\rangle|^2 \times \{\text{lepton factors}\} \times \{\text{kinematical factors}\}$$

- to determine V_{ud}, need to know $\langle proton|j^{ud}_\mu|neutron\rangle$, the transition matrix element

  determined by the strong (colour-charge) interaction of quarks in the nucleon

The world with 4 flavours

- shrorthand notation $\psi_f(x) \to f$, omitting colour index of quarks,

  two doublets: quarks leptons

  $$\begin{pmatrix} u \\ d \end{pmatrix} \begin{matrix} Q_u = 2/3 \\ Q_d = -1/3 \end{matrix} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{matrix} Q_\nu = 0 \\ Q_e = -1 \end{matrix}$$

- antidoublets: $\begin{pmatrix} \overline{d} \\ \overline{u} \end{pmatrix} \begin{matrix} Q_{\bar{d}} = 1/3 \\ Q_{\bar{u}} = -2/3 \end{matrix} \qquad \begin{pmatrix} \overline{e} \\ \overline{\nu}_e \end{pmatrix} \begin{matrix} Q_{\bar{e}} = 1 \\ Q_{\bar{\nu}} = 0 \end{matrix}$

- the doublet structure related with electroweak gauge theory $SU(2) \times U(1)$ uniting QED (γ) and weak interactions W, Z of the quarks and leptons
- Higgs mechanism provides masses to the quarks and leptons, with H -boson discovered in 2013

  – topics for a separate lecture course

- note the importance of the electron mass generated by Higgs mechanism for atomic and molecular physics and our existence
- the first generation of Standard Model

Discovering the second and third generations

- quarks and leptons with flavours beyond the first generation,
  produced in cosmic ray collisions and at high-energy accelerators,
  were also important in the first moments after Big Bang
- were discovered starting from 1936 (muon) till 1995 (top quark), quite unexpectedly ! V. Weisskopf: "Who ordered that?"
- three generations
  of quarks and leptons

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \begin{matrix} Q_{u,c,t} = 2/3 \\ Q_{d,s,b} = -1/3 \end{matrix} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix} \begin{matrix} Q_\nu = 0 \\ Q_{e,\mu,\tau} = -1 \end{matrix}$$

$$\begin{pmatrix} \overline{d} \\ \overline{u} \end{pmatrix} \begin{pmatrix} \overline{s} \\ \overline{c} \end{pmatrix} \begin{pmatrix} \overline{b} \\ \overline{t} \end{pmatrix} \begin{matrix} Q_{\bar{d},\bar{s},\bar{b}} = 1/3 \\ Q_{\bar{u},\bar{c},\bar{t}} = -2/3 \end{matrix} \qquad \begin{pmatrix} \overline{e} \\ \overline{\nu}_e \end{pmatrix} \begin{pmatrix} \overline{\mu} \\ \overline{\nu}_\mu \end{pmatrix} \begin{pmatrix} \overline{\tau} \\ \overline{\nu}_\tau \end{pmatrix} \begin{matrix} Q_{\bar{e},\bar{\mu},\bar{\tau}} = 1 \\ Q_{\bar{\nu}} = 0 \end{matrix}$$

- electroweak (γ, W, Z) interactions are universal for all quark and lepton generations (doublet structure)
  colour-charge interactions universal for all quarks

Quark and lepton masses

- quarks and leptons with different flavour have different masses
- in Standard Model the quark and lepton masses are generated by Higgs mechanism: schematically, for a single flavour

$$\mathcal{L}^f(x) = \lambda_f H(x)\overline{\psi}_f(x)\psi_f(x) = m_f\overline{\psi}_f(x)\psi(x) + \lambda_f h(x)\overline{\psi}_f(x)\psi(x)$$

$H(x) = v + h(x)$, $v \simeq 256$ GeV - vacuum average of the H field, $m_f = \lambda_f v$

- λ_f - nonuniversal Yukawa coupling, specific for the flavour f,
  Higgs mechanism does not allow to predict the masses of quarks and leptons
- quark mass hierarchy
  $m_u \sim 3\,MeV < m_d < m_s \ll m_c < m_b \ll m_t \sim 170$ GeV

Quark mixing matrix

- in the absence of Higgs mechanism:
  three elementary weak transitions $U \to DW$,
  where $U = u, c, t$ and, respectively $D = d, s, b$

- Higgs mechanism in the most general form allowed in the electroweak theory of Standard Model with 3 quark doublets:
  - quark states u, c, t and d, s, b with definite mass;
  - $U \to DW$ flavour changing transitions are not anymore pure $u \to dW$ or $c \to sW$ or $t \to bW$, but their mixture

- the resulting weak current after Higgs mechanism:

$$j_W = (\bar{u} \;\; \bar{c} \;\; \bar{t})\gamma_\mu(1-\gamma_5)\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}\begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

- emerging Cabibbo-Kobayashi-Maskawa (CKM) matrix of flavour-changing transitions; with unitarity property: $VV^+ = I$
- lepton flavours, analogous Pontecorvo -Maki-Nakagawa-Sakata (PMNS) matrix

Consequences of CKM

- rich set of quark flavour transitions in a form of weak decays
  e.g., analogs of β-decay, $s \to u\ell\nu_\ell$, $c \to s\ell\nu_\ell$, $b \to c\ell\nu_\ell$, $(\ell = e, \mu)$
- main properties of CKM-Matrix
  - unitarity relations (test of 3-generations)
  - presence of one complex phase $\Rightarrow$ CP-violation
  - quasidiagonal
- fundamental questions remain unanswered:
  - why three generations?
  - interactions/forces (local gauge symmerties)
    related to flavour quantum numbers? unification of flavour?
  - are quark masses related to CKM parameters?
- many scenarios of new physics beyond Standard Model suggested and being probed by experiments,
- major practical task: accurate determination of quark masses and CKM parameters, tests of unitarity

Importance of quark-gluon interactions

- quarks exist within hadrons (baryons and mesons)

  $\Rightarrow$ to determine quark flavour parameters we need to calculate the effects of colour-charge (quark-gluon) interactions

  (to be discussed in lectures 2,3)

- How important are these interactions?
  - the measured proton and neutron masses:

    m_p=938.272 MeV < m_n=939.565 MeV, (units $\hbar = c = 1$)
  - simple mass formula:

    $m_p = 2m_u + m_d + E_{int}, \quad m_n = 2m_d + m_u + E_{int} \quad (m_d > m_u\ !)$
  - Higgs mechanism generated $m_{u,d} \sim$ a few MeV
  - the quark-gluon interaction energy $E_{int} \sim 99\% m_{p,n}$!,

  $\Longrightarrow$ 99% of the baryon mass in the Universe is due to quark-gluon interactions