Introduction to physics of quarks with flavour and colour

Alexander Khodjamirian

UNIVERSITÄT SIEGEN
Theoretische Physik 1

DFG
RESEARCH UNIT
qfet

Regional Training Network in Theoretical Physics

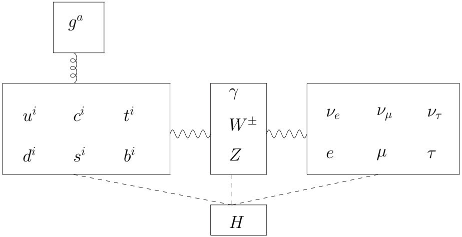
Lectures at the Physics Faculty, Yerevan State University, March 30- April 1 2015

Lecture 2:

Calculating the colour-charge interaction in quantum chromodynamics

$i = 1, 2, 3,$

$a = 1, 2, \ldots, 8$



Colour charge

- "Colour", a dynamical charge carried by quarks
  (in addition to electric charge)

- The colour charge has three components:

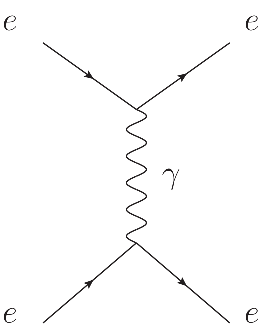
$$u^i = \{u^1, u^2, u^3\} \quad c^i = \{c^1, c^2, c^3\} \quad t^i = \{t^1, t^2, t^3\}$$
$$d^i = \{d^1, d^2, d^3\} \quad s^i = \{s^1, s^2, s^3\} \quad b^i = \{b^1, b^2, b^3\}$$

- three states of the quark with the same flavour, Q, mass

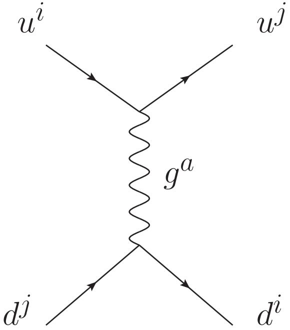
- leptons are colour-neutral

Gluon

- massless particle emitted/absorbed by colour charges
  (photon - massless particle emitted/absorbed by electrical charges)
- Feynman graphs:



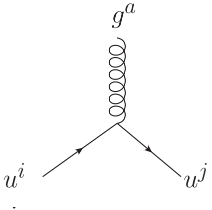
electromagnetic interaction



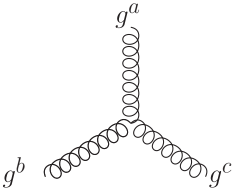
quark-gluon interactions

- gluon carries colour, 8 coloured states:
  $(i = 1, 2, 3) \times (j = 1, 2, 3) \rightarrow a = 1, ..8$

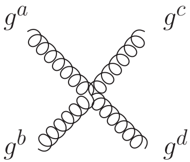
Gluons are self-interacting



quark-gluon vertex



3-gluon vertex



4-gluon vertex

- photons are not self-interacting, light does not emit light !
- gluon self-interactions → confinement of colour charges: free quarks and gluons not observable
- only colour-neutral hadrons (e.g., protons and neutrons) are observable, quarks and gluons are confined inside hadrons
- quark-gluon interaction is flavour neutral: the same coupling g_s for q=u,d,s,..

Quantum Chromodynamics (QCD)

- Quantum field theory of quarks (spin 1/2, Dirac fields), and gluons (spin 1, Maxwell fields)
- QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{q=u,d,s,c,b,t} \sum_k \bar{\psi}^k{}_q (iD_\mu \gamma^\mu - m_q) \psi^k_q \,,$$
$$= \mathcal{L}_{glue} + \mathcal{L}_{quark} + \mathcal{L}_{int}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu \,, \quad D_\mu = \partial_\mu - ig_s \tfrac{\lambda^a}{2} A^a_\mu .$$

- local gauge transformation

$$\psi^i_q(x) = \begin{pmatrix} \psi^1_q(x) \\ \psi^2_q(x) \\ \psi^3_q(x) \end{pmatrix} ,$$

$$\psi^i_q(x) \to \psi'^i_q(x) = U^i_k(x) \psi^k_q(x)$$
$$\bar{\psi}_{q\,i}(x) \to \bar{\psi}'_{q\,i}(x) = \bar{\psi}_k U^{\dagger\,k}_i(x) \,,$$
$$\tfrac{\lambda^a}{2} A^a_\mu(x) \to U(x) \tfrac{\lambda^a}{2} A^a_\mu(x) U^\dagger(x) - \tfrac{i}{g_s} \partial_\mu U(x) U^\dagger(x) \,,$$

Rotations of "colour coordinates"

- 3×3 matrix $U^i_k(x)$, $U^\dagger U = 1$.

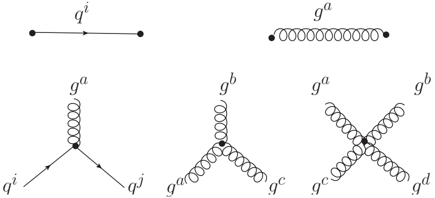
$$U^i_k(x) = \exp\left[-i \sum_{a=1}^{8} \chi^a(x) \frac{(\lambda^a)^i_k}{2} \right] .$$

- eight independent and arbitrary functions $\chi^a(x)$
- Gell-Mann matrices

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix},$$
$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \quad (1)$$

QCD Feynman graphs

- quark-gluon interactions in terms of Feynman graphs:



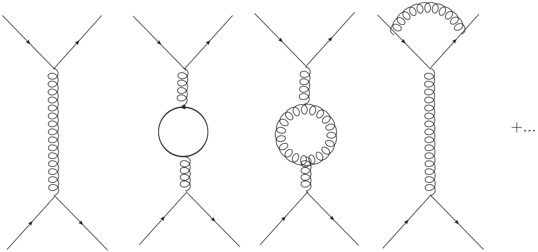
Propagators

Vertices

- $\alpha_s = g_s^2/4\pi$ - dimensionless quark-gluon coupling, analog of $\alpha_{em} = e^2/4\pi$ in QED

Asymptotic freedom of QCD

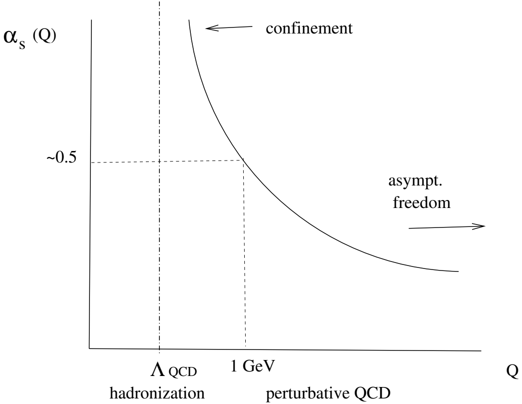
- Quark-quark scattering, energy/momentum transfer Q



- the quantum loop corrections play a crucial role:
  $\alpha_s \to \alpha_s(Q)$, effective, scale-dependent coupling
- $\alpha_s(Q)$ small for processes with $Q \geq 1$ GeV
  $\Rightarrow$ expansion in powers of α_s applicable
- Perturbative QCD: physics of quark and gluon jets
- large momentum transfers $\Rightarrow$ short distances
  (Heisenberg: $\Delta x \Delta p \sim 1$, $\Delta x \sim 1/Q$)

QCD at long distances

- $\alpha_s(Q) \to \infty$ at small momenta $\sim$ long distances



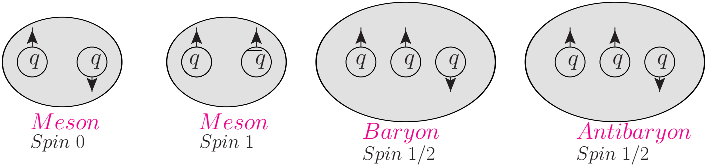
- an intrinsic scale emerges: $\Lambda_{QCD} \sim 200 - 300$ MeV
- at $Q \sim \Lambda_{QCD}$ quarks/gluons strongly interact, hadronization, confinement

QCD Vacuum

- the lowest energy state, no hadrons
  contains fluctuating quark-antiquark and gluon fields:
  vacuum condensates
- e.g., $\langle 0|\overline{q}q|0\rangle \neq 0$, $\quad q = u, d, s$
  -spontaneous breaking of chiral symmetry

The hadron spectroscopy

- Hadrons = colour-neutral bound states



$$|Meson\rangle = \frac{1}{\sqrt{3}} \sum_{i=1,2,3} |\bar{q}_i q^i\rangle \quad |Baryon\rangle = \frac{1}{\sqrt{6}} \sum_{i,j,k=1,2,3} \epsilon_{ijk} |q^i q^j q^k\rangle$$

lowest states: $L = 0$, $J = S$

- various flavour content, $q \to u, d, s, c, b$
- proton and neutron:

$$|proton\rangle = \frac{1}{\sqrt{6}} \sum_{i,j,k=1,2,3} \epsilon_{ijk} |u^i u^j d^k\rangle, \quad |neutron\rangle = \frac{1}{\sqrt{6}} \sum_{i,j,k=1,2,3} \epsilon_{ijk} |u^i d^j d^k\rangle$$

antisymmetry, Pauli-principle

- excited states observed for each spin/flavour combination

- Pseudoscalar mesons ($J = 0^-$), some examples, decay weakly

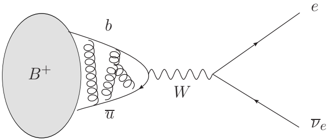
Meson	$\pi^+(140)$	$K^+(494)$	$D^+(1869)$	$B^+(5279)$	$B^0(5279)$
Flavour content	$\|u\bar{d}\rangle$	$\|u\bar{s}\rangle$	$\|c\bar{d}\rangle$	$\|b\bar{u}\rangle$	$\|b\bar{d}\rangle$

(the mass in MeV)

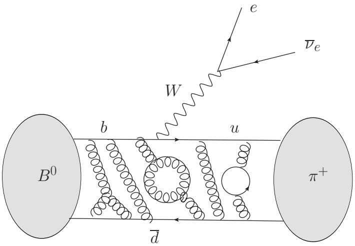
- vector mesons ($J^P = 1^-$) etc. are heavier
  see data in [Particle Data Group], www.pdg.gov
- mesons are generally simpler than baryons
- our main task:
  determination of SM parameters from data on hadrons $\oplus$ QCD
- meson masses $\oplus$ QCD $\rightarrow$ quark masses
- $J^P = O^-$ meson flavour-changing weak decays $\oplus$ QCD $\rightarrow$ V_{CKM}

Flavour-changing B-meson decays

- leptonic mode: decay amplitude $\sim V_{ub}\langle 0|j_W^\mu|B\rangle$, $j_W^\mu = \bar{u}\gamma^\mu(1-\gamma_5)b$



- semileptonic mode: decay amplitude $\sim V_{ub}\langle \pi|j_W^\mu|B\rangle$



- the knowledge of hadronic matrix elements:
  $\langle 0|j_W^\mu|B\rangle$ (decay constant) and $\langle \pi|j_W^\mu|B\rangle$ (form factor)
  allows to extract V_{ub} from exp.data

Hadrons in QCD

- no perturbation theory can be used: $\alpha_s \to \infty$

- quantum fluctuations in hadrons (influence of QCD vacuum): quark-antiquark pairs and gluons in addition to "valence" quarks

  $\Rightarrow$ no interaction potential can be derived

  $\Rightarrow$ no "wave function" of quark constituents in hadron

  $\Rightarrow$ direct calculation of hadron characteristics
  (masses, decay constants, form factors)
  in QCD is currently impossible

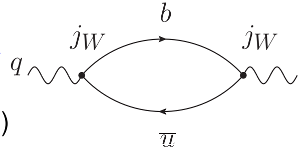
- nonperturbative QCD and formation of hadrons vs other “strong coupling” effects in physics (turbulence, high T_c)

- approximate approaches to nonperturbative QCD developed, with improvable accuracy, based on correlation functions of quark currents

Vacuum correlation function

- amplitude of emission and absorbtion of a $\overline{u}b$ pair in vacuum by $j_W = \bar{u}\gamma_5 b$ operators:

$$\Pi(q^2) = \int d^4x\, e^{iqx}\langle 0|T\{j_W(x)j_W^\dagger(0)\}|0\rangle$$



- rigorously defined ("inverted S-matrix")
- related to the hadronic matrix elements (see lecture 3)

- lattice QCD:
  relating $\Pi(q^2)$ to the functional integral over QCD action and quark-gluon fields;
  space-time → Euclidean space → 4-dim. lattice;
  functional integral → multidimensional integral on the lattice;
  calculated with MC simulation of random ensembles of gauge fields

- we will use a "non-lattice" method: approximate analytical calculation of the correlation function (QCD sum rules)