Introduction to physics of quarks with flavour and colour

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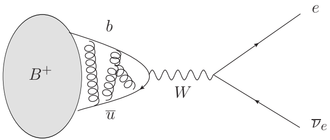
Lecture 3:

Determination of quark masses and mixing parameters



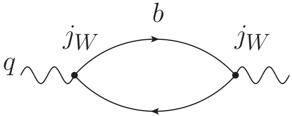
decay amplitude $\sim V_{ub}\langle 0|\bar{u}\gamma_\mu\gamma_5 b|B\rangle$

to calculate hadronic matrix element

Vacuum correlation function

- amplitude of emission and absorbtion of a $\overline{u}b$ pair in vacuum by $j_W = \bar{u}\gamma_5 b$ currents: (using Dirac eq. : $\partial^\mu(\bar{u}\gamma_\mu\gamma_5 b) = (m_b + m_u)j_W$)

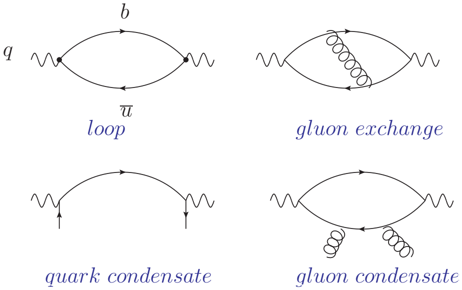
$\Pi(q^2) = \int d^4x\, e^{iqx}\langle 0|T\{j_W(x)j_W^\dagger(0)\}|0\rangle$



- The 4-momentum $q = (q_0, \vec{q})$ of the quark-antiquark pair: rest frame: $\vec{q} = 0$, $q^2 = q_0^2$, fix the energy $q_0 \ll m_b$
- the $\overline{u}b$-pair is virtual: $\Delta E \Delta t \sim 1$, the energy deficit $\Delta E \sim m_b$, $\Rightarrow$ the quarks propagate during a short time: $\Delta t \sim 1/m_b$
- the scale $m_b \gg \Lambda_{QCD}$: quarks asymptotically free, a loop diagram

Calculating the correlation function

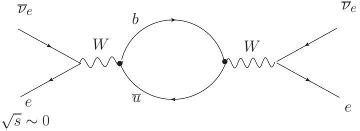
- adding effects of gluon exchange within loop $\alpha_s(m_B) \ll 1$
- including nonperturbative effects due to condensates

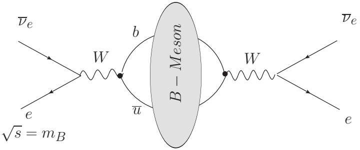


- Result: analytical expression for $\Pi(q^2)$ in terms of m_b,m_u and universal QCD parameters α_s, $\langle 0|\bar{q}q|0\rangle$,$\langle 0|GG|0\rangle$

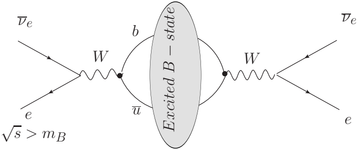
Relating correlation function with hadrons

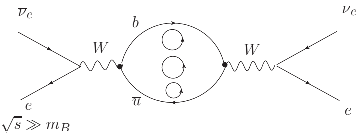
- Hypothetical neutrino-electron scattering, varying c.m. energy $\sqrt{s} = \sqrt{q^2}$,
- $\Pi(q^2)$ is the part of the scattering amplitude





highly virtual quark pair, B-meson, resonance





excited B mesons multiple hadrons (continuum)

- $\Pi(q^2)$ can be represented as a hadronic sum

QCD sum rule

- Hadronic sum (dispersion relation)

$$\Pi(q^2) = \frac{\langle 0|j_W|B\rangle\langle B|j_W|0\rangle}{m_B^2 - q^2} + \sum_{B_{exc},\dots} \frac{\langle 0|j_W|B_{exc}\rangle\langle B_{exc}|j_W|0\rangle}{m_{B_{exc}}^2 - q^2}$$

- derivation based on the unitarity and analyticity of scattering amplitudes in quantum field theory
- using the QCD result for the correlation function:

$$\Pi_{QCD}(q^2; \alpha_s, m_b, m_u, <cond>) = \text{Hadronic} \quad \text{Sum}$$

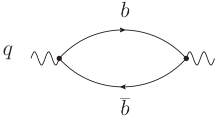
$$\rightarrow \text{QCD sum rule}, \text{ valid at } q^2 \ll m_B^2$$

- a general method, applicable for various local quark operators and correlation functions
- input: $\alpha_s(m_Z) = 0.1176 \pm 0.002 \rightarrow \alpha_s(m_B)$ (perturbative QCD),

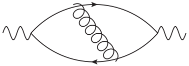
Determination of b-quark mass

- change the current in the correlation function: $j_W \to \bar{b}\gamma_\nu b$

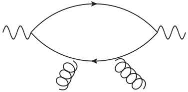
no quark condensate,
only heavy quarks



loop


gluon exchange

- the hadronic states are Υ-meson and excited $\bar{b}b$ states, well measured in e^+e^- collisions $\Rightarrow$ hadronic sum


gluon condensate

- fitting m_b and gluon condensate from the sum rule

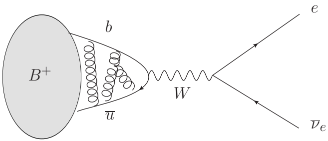
$$\Pi^{\bar{b}b}(q^2, \alpha_s, m_b, \langle GG \rangle) = \frac{\langle 0|\bar{b}\gamma_\mu b|\Upsilon\rangle\langle\Upsilon|\bar{b}\gamma^\mu b|0\rangle}{m_\Upsilon^2 - q^2} + \sum_{\Upsilon_{exc}} \frac{\langle 0|\bar{b}\gamma_\mu b|\Upsilon_{exc}\rangle\langle\Upsilon_{exc}|\bar{b}\gamma^\mu b|0\rangle}{m_{\Upsilon_{exc}}^2 - q^2}$$

- scale-dependence of the quark mass,
  $m_b(Q = m_b) = 4.18 \pm 0.03$ GeV
- analog c-quark from charmonium sum rules:
  $m_c(Q = m_c) = 1.275 \pm 0.025$ GeV

Quark condensate and light quark masses

- correlation functions of light-quark currents $j_\mu^{ud} = \bar{u}\gamma_\mu(1-\gamma_5)d$
- input: masses of π-meson, hadronic matrix element $\langle 0|j^{ud}|\pi\rangle$ from $\pi \to \mu\nu_\mu$ decay (V_{ud} from nuclear β decays)
- the sum rules for strange-quark currents
- results (PDG averages), lattice and non-lattice QCD,
  with comparable errors:
  $m_u(Q=2GeV) = 2.15 \pm 0.15$ MeV,
  $m_d(Q=2GeV) = 4.70 \pm 0.20$ MeV,
  $m_s(Q=2GeV) = 93.5 \pm 2.5$ MeV.
- quark condensate density
  $\langle 0|\bar{q}q|0\rangle(Q=2GeV) = (-277^{+12}_{-10}\ MeV)^3$
  input fixed for the sum rule for B-meson hadronic matrix element

Calculation of the B meson decay constant



decay constant $\sim \langle 0|j_W|B\rangle \equiv f_B m_B^2$

- QCD sum rule (schematically)

$$\Pi_{QCD}(q^2;\alpha_s, m_b, m_u, <\bar{q}q>, ..) = \frac{\langle 0|j_W|B\rangle\langle B|j_W|0\rangle}{m_B^2 - q^2} + \left\{\sum_{B_{exc}}\right\}_{duality\ approx.}$$

- our recent result obtained with NNLO accuracy:
  [P.Gelhausen, D.Rosenthal, AK, A.Pivovarov, Phys.Rev. D88 2013]

$$f_B = 207^{+17}_{-9}\ \text{MeV}$$

lattice QCD: $f_B = 197 \pm 9$ MeV [Fermilab-MILC Collaboration, 2012]

Extraction of $|V_{ub}|$

- leptonic decay amplitude:
  $A(B^- \to \tau^- \bar{\nu}_\tau)_{SM} = \frac{G_F}{\sqrt{2}} V_{ub} \langle 0|\bar{u}\gamma_\mu\gamma_5 b|B\rangle \,\bar{\tau}\gamma^\mu(1-\gamma_5)\nu_\tau$

- partial width: (suppressed for $\ell = \mu, e$)

$$BR(B^- \to \tau^- \bar{\nu}_\tau)_{SM} = \frac{G_F^2 |V_{ub}|^2}{8\pi} m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 \tau_{B^-} \,,$$

- the most accurate measurement [Belle collaboration (2015)]

$$BR(B \to \tau\nu_\tau) = (1.25 \pm 0.28(stat.) \pm 0.27(syst.)) \times 10^{-4}$$

$$\Rightarrow |V_{ub}| = (4.05 \pm 0.39_{exp} [^{+0.18}_{-0.31}]_{f_B}) \times 10^{-3}$$

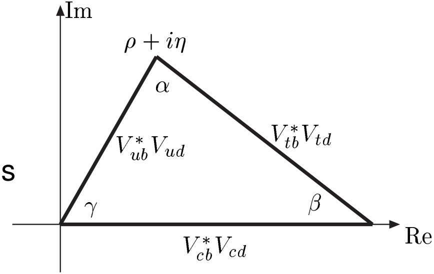
- currently, more accurate $|V_{ub}|$ from semileptonic $B \to \pi\ell\nu_\ell$
- 3σ discrepancy with "inclusive" determination from $B \to X_u \ell\nu_\ell$

V_{CKM} parametrizations and phase

- Unitarity of V_{CKM} , $\lambda \sim sin\,\theta_C$,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

- α, β, γ
  (over)determined from CP-violating asymmetries in selected nonleptonic B-decays
  (e.g., $B \to J/\psi K \to \beta$)



Summary on $|V_{CKM}|$ determination

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ \\ V_{cd} & V_{cs} & V_{cb} \\ \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \Leftrightarrow \begin{pmatrix} \begin{matrix} 0^+ \to 0^+ \\ \text{nuclear}\,\beta \end{matrix} & \begin{matrix} K \to \ell\nu_\ell \\ K \to \pi\ell\nu_\ell \end{matrix} & \begin{matrix} B \to \ell\nu_\ell \\ B \to \pi\ell\nu_\ell \\ B \to X_u\ell\nu_\ell \end{matrix} \\ \\ \begin{matrix} D \to \ell\nu_l \\ D \to \pi\ell\nu_l \end{matrix} & \begin{matrix} D_s \to \ell\nu_\ell \\ D \to K\ell\nu_\ell \end{matrix} & \begin{matrix} B \to D^{(*)}l\nu_\ell \\ B \to X_c\ell\nu_\ell \end{matrix} \\ \\ B_d \leftrightarrow \bar{B}_d & B_s \leftrightarrow \bar{B}_s & t \to bW \end{pmatrix}$$

- V_{CKM} determination needs hadronic matrix elements
- two exceptions: $|V_{ud}| = 0.97418 \pm 0.00027$

  (superallowed nuclear β transitions, CVC, updated in 2009)

  $|V_{tb}| > 0.74$ (single t-quark decays, CDF+DØ)
  $|V_{tb}| = 0.77^{+0.18}_{-0.24}$ (preicision EW)

Some further reading

available online from http://inspirehep.net/

- Y.Grossman , Introduction to flavor physics 1006.3534 [hep-ph] (2010)
- Andrzej J. Buras , Flavour Physics and CP Violation hep-ph/0505175 (2005)
- A. Khodjamirian, ‘Quantum chromodynamics and hadrons: An Elementary introduction,” hep-ph/0403145. (2004)